Allocating data for broadcasting over wireless channels subject to transmission errors

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Abstract

Broadcasting is an efficient and scalable way of transmitting data over wireless channels to an unlimited number of clients. In this paper the problem of allocating data to multiple channels is studied, assuming flat data scheduling per channel and the presence of unrecoverable channel transmission errors. The objective is that of minimizing the average expected delay experienced by the clients. Two different channel error models are considered: the Bernoulli model and the simplified Gilbert-Elliot one. In the former model, each packet transmission has the same probability to fail and each transmission error is independent from the others. In the latter one, bursts of erroneous or error-free packet transmissions due to wireless fading channels are modeled. For both channel error models and unit length data, an optimal solution can be found in polynomial time either when all the channels have the same error probabilities or when there are exactly two channels with different error probabilities. In the remaining cases, including non-unit length data and error probabilities which differ from channel to channel, sub-optimal solutions can be found for both error models. For these cases, extensive simulations, performed on benchmarks whose item popularities follow Zipf distributions, show that good sub-optimal solutions are found.

Keywords Wireless communication, Data broadcasting, Multiple channels, Flat scheduling, Average expected delay, Channel transmission errors, Bernoulli model, Gilbert-Elliot model, Heuristics.

1 Introduction

In wireless asymmetric communications, broadcasting is an efficient way of simultaneously disseminating data to a large number of clients [17]. Consider data services on cellular

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networks, such as stock quotes, weather infos, traffic news, where data are continuously broadcast to clients that may desire them at any instant of time. In this scenario, a server at the base-station repeatedly transmits data items from a given set over wireless channels, while clients passively listen to the shared channels waiting for their desired item. The server has to pursue a data allocation strategy for assigning items to channels and a broadcast schedule for deciding which item has to be transmitted on each channel at any time instant. Efficient data allocation and broadcast scheduling have to minimize the client expected delay, that is, the average amount of time spent by a client before receiving the item he needs. The client expected delay increases with the size of the set of the data items to be transmitted by the server. Indeed, the client has to wait for many unwanted data before receiving his own data. Moreover, the client expected delay may be influenced by transmission errors because items are not always received correctly by the client. Although data are usually encoded using *error correcting codes* (*ECC*) allowing some recoverable errors to be corrected by the client without affecting the average expected delay, there are several transmission errors which still cannot be corrected using ECC. Such *unrecoverable* errors affect the client expected delay, because the resulting corrupted items have to be discarded and the client must wait until the same item is broadcast again by the server.

Several variants for the problem of data allocation and broadcast scheduling have been proposed in the literature [1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 13, 15, 16, 19, 21, 22].

The database community usually partitions the data among the channels and then adopts a *flat* broadcast schedule on each channel [5, 15, 22]. In such a way, the allocation of data to channels becomes critical for reducing the average expected delay, while the flat schedule on each channel merely consists in cyclically broadcasting in an arbitrary fixed order, that is once at a time in a round-robin fashion, the items assigned to the same channel [1]. To reduce the average expected delay, *skewed* data allocations are used, where items are partitioned according to their popularities so that the most requested items appear in a channel with shorter period. Assuming that each item transmitted by the server is always received correctly by the client, a solution that minimizes the average expected delay can be found in polynomial time in the case of *unit lengths* [22], that is when the transmission time is equal to one for all items, whereas the problem becomes computationally intractable for non-unit lengths [5]. In this latter case, several heuristics have been developed in [4, 22], which have been tested on some benchmarks where item popularities follow Zipf distributions. Such distributions are used to characterize the popularity of one item among a set of similar data, like a web page in a web site [8].

Thus far, the data allocation problem has not been investigated by the database community when the wireless channels are subject to transmission errors. In contrast, a wireless environment subject to errors has been considered by the networking community, which however concentrates only on finding broadcast scheduling for a single channel to minimize the average expected delay [6, 10, 11, 19]. Indeed, the networking community assumes all items replicated over all channels, and therefore no data allocation to the channels is needed. Although it is still unknown whether a broadcast schedule on a single channel with minimum average delay can be found in polynomial time or not, almost all the proposed solutions follow the *square root rule (SRR)*, a heuristic which in practice finds near-optimal schedules [3]. The aim of SRR is to produce a broadcast schedule where each data item appears with equally spaced replicas, whose frequency is proportional to the square root of its popularity and inversely proportional to the square root of its length. In particular, the solution proposed by [19] adapts the SRR solution to the case of unrecoverable errors. In such a case, since corrupted items must be discarded worsening the average expected delay, the spacing among replicas has to be properly recomputed.

The present paper extends the data allocation problem first studied by the database community under the assumptions of multiple channels and flat data schedule per channel [4, 5, 22], to cope with the presence of erroneous transmissions, under the same assumptions of [19], namely unrecoverable errors. Two different error models will be considered to describe the behavior of wireless channels [20]. First, as in [19], the Bernoulli channel error is assumed, where each packet transmission has the same probability q to fail and $1 - q$ to succeed, and each transmission error is independent from the others. Then, the so called simplified Gilbert-Elliot channel error model will be considered, which was not previously studied in [19]. Such a model is able to capture burstiness, that is sequences of erroneous or error-free packet transmissions, and well approximates the error characteristics of certain wireless fading channels [18, 23]. As in [19], the erroneous transmissions are taken into account in the problem parameters and they are compensated by properly modifying the allocation of data items to channels. Specifically, for both channel error models, it will be shown that an optimum solution, namely one minimizing the average expected delay, can be found in polynomial time for the data allocation problem when the data items have unit lengths and all the channels have the same error probability. When such a probability differs from channel to channel, sub-optimal solutions will be found for both unit and non-unit length data and both Bernoulli and Gilbert-Elliot error models. However, when there are exactly two channels, an optimal solution can be found in polynomial time for both error models, unit length data, and different channel error probabilities, and in pseudo-polynomial time for the Bernoulli model, non-unit length data, and channels with the same error probability. Extensive simulations will show that good sub-optimal solutions are found on benchmarks whose items probabilities are characterized by Zipf distributions.

The rest of this paper is so organized. Section 2 first gives notations, definitions as well as the problem statement, and then recalls the basic dynamic programming algorithms known so far in the case of error-free channel transmissions. Sections 3 and 4 consider the Bernoulli and the Gilbert-Elliot channel error models, respectively, and illustrate heuristics for items of unit and non-unit lengths. Such heuristics are derived by properly redefining the recurrences in the dynamic programming algorithms previously presented for error-free channels. In particular, Sections 3 and 4 also present the optimal algorithms for the above mentioned special cases. Experimental tests are reported at the end of both Sections 3 and 4. Finally, conclusions are offered in Section 5.

2 Background on error-free channels

Consider a set of K identical error-free channels, and a set $D = \{d_1, d_2, \ldots, d_N\}$ of N data items. Each item d_i is characterized by a *popularity* p_i and a *length* z_i, with $1 \leq i \leq N$. The popularity p_i represents the demand probability of item d_i, namely its probability to be requested by the clients, and it does not vary along the time. Clearly, $\sum_{i=1}^{N} p_i = 1$. The length z_i is an integer number, counting how many packets are required to transmit item d_i on any channel and it includes the encoding of the item with an error correcting code. For the sake of simplicity, it is assumed that a packet transmission requires one time unit. Each d_i is assumed to be non preemptive, that is, its transmission cannot be interrupted. When all data lengths are unitary, i.e., $z_i = 1$ for $1 \leq i \leq N$, the lengths are called *unit* lengths, otherwise they are said to be *non-unit* lengths.

The *expected delay* t_i is the expected number of packets a client must wait for receiving item d_i. The *average expected delay* (AED) is the number of packets a client must wait on the average for receiving any item, and is computed as the sum over all items of their expected delay multiplied by their popularity, that is

$$\text{AED} = \sum_{i=1}^{N} t_i p_i \tag{1}$$

When the items are partitioned into K groups $G_1, \ldots, G_K$, where group G_k collects the data items assigned to channel k, and a flat schedule is adopted for each channel, that is, the items in G_k are cyclically broadcast in an arbitrary fixed order, Equation 1 can be simplified. Indeed, if item d_i is assigned to channel k, and assuming that clients can start to listen at any instant of time with the same probability, then t_i becomes $\frac{Z_k}{2}$, where Z_k is the schedule *period* on channel k, i.e., $Z_k = \sum_{d_i \in G_k} z_i$. Then, Equation 1 can be rewritten as

$$\text{AED} = \sum_{i=1}^{N} t_i p_i = \sum_{k=1}^{K} \sum_{d_i \in G_k} \frac{Z_k}{2} p_i = \sum_{k=1}^{K} \left(\frac{Z_k}{2} \sum_{d_i \in G_k} p_i \right) = \frac{1}{2} \sum_{k=1}^{K} Z_k P_k \tag{2}$$

where P_k denotes the sum of the popularities of the items assigned to channel k, i.e., $P_k =$

$\sum_{d_i \in G_k} p_i$. Note that, in the unit length case, the period Z_k coincides with the cardinality of G_k, which will be denoted by N_k.

Summarizing, given K error-free channels, a set D of N items, where each data item d_i comes along with its popularity p_i and its integer length z_i, the *Data Allocation Problem* consists in partitioning D into K groups $G_1, \ldots, G_K$, so as to minimize the AED objective function given in Equation 2. Note that, in the special case of unit lengths, the corresponding objective function is derived replacing Z_k with N_k in Equation 2.

Almost all the algorithms proposed so far for the data allocation problem on error-free channels are based on dynamic programming. Such algorithms restrict the search for the solutions to the so called *segmentations*, that is, partitions obtained by considering the items ordered by their indices, and by assigning items with consecutive indices to each channel. Formally, a segmentation is a partition of the ordered sequence $d_1, \ldots, d_N$ into K adjacent segments $G_1, \ldots, G_K$, each of consecutive items, as follows:

$$\underbrace{d_1, \ldots, d_{B_1}}_{G_1}, \underbrace{d_{B_1+1}, \ldots, d_{B_2}}_{G_2}, \ldots, \underbrace{d_{B_{K-1}+1}, \ldots, d_N}_{G_K}$$

A segmentation can be compactly denoted by the $(K-1)$-tuple

$$(B_1, B_2, \ldots, B_{K-1})$$

of its *right borders*, where border B_k is the index of the last item that belongs to group G_k. Notice that it is not necessary to specify B_K, the index of the last item of the last group, because its value will be N for any solution.

Four main dynamic programming algorithms for the data allocation problem are now briefly surveyed, called *DP*, *Dichotomic*, *Dlinear*, and *Knapsack*. All the algorithms, except the last one, assume that the items $d_1, d_2, \ldots, d_N$ are indexed by non-increasing $\frac{p_i}{z_i}$ ratios, that is $\frac{p_1}{z_1} \geq \frac{p_2}{z_2} \geq \cdots \geq \frac{p_N}{z_N}$. Observe that in the unit length case this means that the items are sorted by non-increasing popularities. Let $SOL_{k,n}$ denote a segmentation for grouping items $d_1, \ldots d_n$ into k groups and let $sol_{k,n}$ be its corresponding cost, for any $k \leq K$ and $n \leq N$. Moreover, let $C_{i,j}$ denote the cost of assigning to a single channel the consecutive items $d_i, \ldots, d_j$:

$$C_{i,j} = \sum_{h=i}^{j} t_h p_h = \sum_{h=i}^{j} \left(\frac{1}{2} \sum_{h=i}^{j} z_h \right) p_h = \frac{1}{2} \left(\sum_{h=i}^{j} z_h \right) \left(\sum_{h=i}^{j} p_h \right) \tag{3}$$

For unit lengths, the above formula simplifies as $C_{i,j} = \frac{1}{2}(j - i + 1) \sum_{h=i}^{j} p_h$. Note that, once the items are sorted, all the $C_{i,j}$'s can be found in $O(N)$ time by means of prefix-sum computations [21].

The DP algorithm is a dynamic programming implementation of the following recurrence, where k varies from 1 to K and, for each fixed k, n varies from 1 to N:

$$sol_{k,n} = \begin{cases} C_{1,n} & \text{if } k = 1 \\ \min_{1 \leq \ell \leq n-1}\{sol_{k-1,\ell} + C_{\ell+1,n}\} & \text{if } k > 1 \end{cases} \tag{4}$$

For any value of k and n, the DP algorithm selects the best solution obtained by considering the $n-1$ solutions already computed for the first $k-1$ channels and for the first ℓ items, and by combining each of them with the cost of assigning the last $n-\ell$ items to the single k-th channel. The DP algorithm requires $O(N^2K)$ time. It finds an optimal solution in the case of unit lengths and a sub-optimal one in the case of non-unit lengths [22].

To improve on the time complexity of the DP algorithm, the Dichotomic algorithm has been devised. Let B_h^n denote the h-th border of $SOL_{k,n}$, with $k > h \geq 1$. Assume that $SOL_{k-1,n}$ has been found for every $1 \leq n \leq N$. If $SOL_{k,l}$ and $SOL_{k,r}$ have been found for some $1 \leq l \leq r \leq N$, then one knows that B_{k-1}^c is between B_{k-1}^l and B_{k-1}^r, for any $l \leq c \leq r$. Thus, choosing c as the middle point between l and r, Recurrence 4 can be rewritten as:

$$sol_{k,\lceil\frac{l+r}{2}\rceil} = \min_{B_{k-1}^l \leq \ell \leq B_{k-1}^r} \{sol_{k-1,\ell} + C_{\ell+1,\lceil\frac{l+r}{2}\rceil}\} \tag{5}$$

where B_{k-1}^l and B_{k-1}^r are, respectively, the final borders of $SOL_{k,l}$ and $SOL_{k,r}$. The Dichotomic algorithm reduces the time complexity of the DP algorithm to $O(NK\log N)$. As for the DP algorithm, the Dichotomic algorithm also finds optimal and sub-optimal solutions for unit and non-unit lengths, respectively [5].

Moreover, fixed k and n, the Dlinear algorithm selects the feasible solutions that satisfy the following Recurrence:

$$sol_{k,n} = \begin{cases} C_{1,n} & \text{if } k = 1 \\ sol_{k-1,m} + C_{m+1,n} & \text{if } k > 1 \end{cases} \tag{6}$$

where

$$m = \min_{B_k^{n-1} \leq \ell \leq n-1} \{\ell \ : sol_{k-1,\ell} + C_{\ell+1,n} < sol_{k-1,\ell+1} + C_{\ell+2,n}\}\,.$$

In practice, Dlinear adapts Recurrence 4 by exploiting the property that, if $SOL_{k,n-1}$ is known, then one knows that B_k^n is no smaller than B_k^{n-1}, and by stopping the trials as soon as the cost $sol_{k-1,\ell} + C_{\ell+1,n}$ of the solution starts to increase. The overall time complexity of the Dlinear algorithm is $O(N(K + \log N))$. Thus the Dlinear algorithm is even faster than the Dichotomic one, but the solutions it provides are always sub-optimal, both in the case of unit and non-unit lengths [4].

The Knapsack algorithm solves the problem when there are exactly 2 channels. In such a case, the problem is to find a partition G_1 and G_2 such that $\frac{1}{2}(Z_1P_1 + Z_2P_2)$ is minimized.

Without loss of generality, $Z_1 \le Z_2$ can be assumed. Observe that $Z_1P_1 + Z_2P_2 = Z_1P_1 + Z_2(1-P_1) = P_1(Z_1-Z_2)+Z_2$. When Z_1 is fixed, also $Z_2 = Z-Z_1$ is fixed, where $Z = \sum_{i=1}^{N} z_i$. Noting that $Z_1 - Z_2 \le 0$, minimizing $Z_1P_1+Z_2P_2$ is equivalent to maximizing P_1. Therefore, the problem reduces to a particular *Knapsack problem* [14] of capacity Z_1, where each item d_i is characterized by a *profit* p_i and a *weight* z_i. Consider an $(N+1)\times(\lfloor Z/2\rfloor+1)$ matrix M. The entry $M_{i,j}$, with $0 \le i \le N$ and $0 \le j \le \lfloor Z/2\rfloor$, stores the value of the objective function for a Knapsack problem with items $\{d_1, \ldots, d_i\}$ and capacity j. Formally, $M_{i,j} = \max \sum_{d_k \in S} p_k$ such that $\sum_{d_k \in S} z_k = j$, where $S \subseteq \{d_1, \ldots, d_i\}$. By definition, $M_{i,j} = -\infty$ if the capacity j cannot be completely filled by any S. The dynamic programming algorithm starts by initializing the first row of M in such a way that $M_{0,0} = 0$, $M_{0,j} = -\infty$ for $1 \le j \le \lfloor Z/2\rfloor$, and by using the following relation:

$$M_{i,j} = \begin{cases} M_{i-1,j} & \text{if } \ j < z_i \\ \max\{M_{i-1,j},\ M_{i-1,j-z_i} + p_i\} & \text{if } \ j \ge z_i \end{cases} \tag{7}$$

Consider the last row of M. Any entry $M_{N,j} \ne -\infty$ gives the optimal P_1 for the 2-channel data allocation problem with items $\{d_1, \ldots, d_N\}$ and $Z_1 = j$. Therefore, the entry, say $M_{N,\bar{j}}$, which minimizes $\frac{1}{2}\left(\bar{j}M_{N,\bar{j}} + (Z-\bar{j})(1-M_{N,\bar{j}})\right)$ gives the optimal AED for the original problem. Once $M_{N,\bar{j}}$ has been found, it is easy to list out the items which have been picked up in the optimal solution, by tracing back the solution path. The Knapsack algorithm always finds an optimal solution for two channels and non-unit lengths and its overall time complexity is $O(NZ)$, which is pseudo-polynomial [5].

3 Bernoulli channel error model

In this section, unrecoverable channel transmission errors modeled by a geometric distribution are taken into account. Under such an error model, each packet transmission over channel k has the same probability q_k to fail and $1 - q_k$ to succeed, and each transmission error is independent from the others, with $1 \le k \le K$ and $0 \le q_k \le 1$. Since the environment is asymmetric, a client cannot ask the server to immediately retransmit an item d_i which has been received on channel k with an unrecoverable error. Indeed, the client has to discard the item and then has to wait for a whole period Z_k, until the next transmission of d_i scheduled by the server. Even the next item transmission could be corrupted, and in such a case an additional delay of Z_k has to be waited. Therefore, the expected delay t_i has to take into account the extra waiting time due to a possible sequence of independent unrecoverable errors.

3.1 Unit length items

Assume that the item lengths are unitary, i.e., $z_i = 1$ for $1 \leq i \leq N$. Recall that in such a case the period of channel k is N_k. If a client wants to receive item d_i, which is transmitted on channel k, and the first transmission he can hear of d_i is error-free, then the client waits on the average $\frac{N_k}{2}$ time units with probability $1 - q_k$. Instead, if the first transmission of d_i is erroneous, but the second one is error-free, then the client experiences an average delay of $\frac{N_k}{2} + N_k$ time units with probability $q_k(1-q_k)$. Generalizing, if there are h bad transmissions of d_i followed by a good one, the client average delay for receiving item d_i becomes $\frac{N_k}{2} + hN_k$ time units with probability $q_k^h(1 - q_k)$. Thus, summing up over all h, the expected delay t_i is equal to

$$\sum_{h=0}^{\infty}(\frac{N_k}{2} + hN_k)q_k^h(1 - q_k) = \frac{N_k}{2}(1 - q_k)\sum_{h=0}^{\infty} q_k^h + N_k(1 - q_k)\sum_{h=0}^{\infty} hq_k^h = \frac{N_k}{2} + N_k\frac{q_k}{1 - q_k}$$

because $\sum_{h=0}^{\infty} q_k^h = \frac{1}{1-q_k}$ and $\sum_{h=0}^{\infty} hq_k^h = \frac{q_k}{(1-q_k)^2}$. Therefore, one can set the expected delay as

$$t_i = \frac{N_k}{2}\frac{1 + q_k}{1 - q_k} \tag{8}$$

By the above setting, the objective function given in Equation 1 can be rewritten as

$$\text{AED} = \sum_{i=1}^{N} t_i p_i = \frac{1}{2}\sum_{k=1}^{K} N_k \frac{1 + q_k}{1 - q_k} P_k \tag{9}$$

Note that the AED to be minimized depends now not only on the items allocated to each group but also on the channel assigned to each group. Hence, Equation 9 represents the new objective function for the problem of allocating data to multiple channels assuming unit length items, flat data scheduling per channel, and unrecoverable channel transmission errors modeled by a geometric distribution.

The following result shows that there is an optimal solution where the items are sorted by non-increasing popularities.

Lemma 1. *Let G_h and G_j be two groups in an optimal solution. Let d_i and d_k be items with $d_i \in G_h$ and $d_k \in G_j$. If $N_h\frac{1+q_h}{1-q_h} < N_j\frac{1+q_j}{1-q_j}$, then $p_i \geq p_k$. Similarly, if $p_i > p_k$, then $N_h\frac{1+q_h}{1-q_h} \leq N_j\frac{1+q_j}{1-q_j}$.*

Proof. By contradiction, let $G_1, G_2, \ldots, G_K$ be an optimal solution for which there exist G_h and G_j such that $N_h\frac{1+q_h}{1-q_h} < N_j\frac{1+q_j}{1-q_j}$ and $p_i < p_k$. Consider now another solution obtained by exchanging d_i with d_k in the two groups G_h and G_j. The difference in the AED of the two solutions is $\left(N_h\frac{1+q_h}{1-q_h} - N_j\frac{1+q_j}{1-q_j}\right)(p_i - p_k) > 0$ because $p_i - p_k < 0$ and $N_h\frac{1+q_h}{1-q_h} - N_j\frac{1+q_j}{1-q_j} < 0$. Hence, a better solution is achieved contradicting the optimality assumption. The last part of the lemma is proved similarly. □

In practice, by the above lemma, an optimal solution which is a segmentation always exists. However, no hints are given on which order the channels have to be taken. Nonetheless, the next result shows that an optimal solution exists where the channels are indexed by non-decreasing channel error probabilities.

Lemma 2. *Let G_h and G_j be two groups in an optimal solution. If $N_hP_h > N_jP_j$, then $q_h \leq q_j$. Similarly, if $q_h < q_j$, then $N_hP_h \geq N_jP_j$.*

Proof. By contradiction, let $G_1, G_2, \ldots, G_K$ be an optimal solution for which there exist G_h and G_j such that $N_hP_h > N_jP_j$ and $q_h > q_j$. Consider now another solution obtained by exchanging the items assigned to the groups G_h and G_j. The difference in the AED of the two solutions is $(N_hP_h - N_jP_j)(\frac{1+q_h}{1-q_h} - \frac{1+q_j}{1-q_j}) > 0$. Indeed $N_hP_h - N_jP_j > 0$ and $\frac{1+q_h}{1-q_h} - \frac{1+q_j}{1-q_j} > 0$ because $q_h > q_j$. Hence, a better solution is achieved contradicting the optimality assumption. The last part of the lemma is proved similarly. □

Unfortunately, an optimal solution which is a segmentation and takes the channels by non-decreasing error probabilities does not always exist, as shown by the following counterexample. Let $N = 7$ and $D = \{d_1, \ldots, d_7\}$ with $p_1 = \frac{5}{6}$, $p_2 = \frac{1}{12}$, $p_3 = \ldots = p_7 = \frac{1}{60}$. Moreover let $K = 3$ where $q_1 = 0, q_2 = \frac{1}{3}$ and $q_3 = \frac{1}{2}$. An optimal solution which is a segmentation assigns d_1 to the channel with error probability q_1, d_2 to that with error probability q_3, and all the remaining items to the channel with probability q_2. Such a solution satisfies Lemma 1, but does not take the channels by non-decreasing error probabilities. However, the above solution can be rearranged in such a way that Lemma 2 holds, obtaining a new optimal solution which assigns d_1 to the channel with error probability q_1, $\{d_3, d_4, d_5, d_6, d_7\}$ to that with error probability q_2, and d_2 to the channel with error probability q_3. Clearly, although this solution takes the channels in non-decreasing error probabilities, it does not maintain items sorted by non-increasing popularities.

In the special case where there are only two channels, an optimal solution can be efficiently found by exploiting the following result.

Corollary 1. *Assume $K = 2$ and the items sorted by non-increasing popularities, and let (B_1) be an optimal segmentation. Then, $B_1 \leq (N - B_1)\frac{1+q_{max}}{1-q_{max}}\frac{1-q_{min}}{1+q_{min}}$, where q_{max} and q_{min} are the largest and the smallest error probabilities, respectively. Moreover, if $B_1 \geq \lceil \frac{N}{2} \rceil$ then the items $d_1, \ldots, d_{B_1}$ are assigned to the channel with error probability q_{min}.*

Proof. By contradiction, let $B_1 > (N - B_1)\frac{1+q_{max}}{1-q_{max}}\frac{1-q_{min}}{1+q_{min}}$. Then $N_1\frac{1+q_{min}}{1-q_{min}} > N_2\frac{1+q_{max}}{1-q_{max}}$. By Lemma 1, the item popularities are non-decreasing contradicting the assumption. To show the remaining property, observe that, since $B_1 \geq \lceil \frac{N}{2} \rceil$ and the items are sorted by non-increasing popularities, then $N_1 \geq N_2$, $P_1 \geq P_2$, and hence $N_1P_1 \geq N_2P_2$. By Lemma 2, the channels must be taken by increasing error probabilities. Therefore, the first group of items will be assigned to the channel with minimum error probability q_{min}. □

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Input:       N items sorted by non-increasing popularities {p_1, p_2, ..., p_N};
             K = 2 channels; channel error probabilities q_min and q_max;
Initialize:  N_1 := 1, N_2 := N - N_1, B_1 := 1;
             P_1 := p_1, P_2 := 1 - P_1;
             if N_1 P_1 > N_2 P_2 then q_1 := q_min, q_2 := q_max
                            else q_1 := q_max, q_2 := q_min;
             AED := N_1 P_1 (1+q_1)/(1-q_1) + N_2 P_2 (1+q_2)/(1-q_2);
Loop 1:      while q_1 > q_2 do begin
                 N_1 := N_1 + 1, N_2 := N_2 - 1;
                 P_1 := P_1 + p_{N_1}, P_2 := P_2 - p_{N_1};
                 if N_1 P_1 > N_2 P_2 then q_1 := q_min, q_2 := q_max;
                 if AED > N_1 P_1 (1+q_1)/(1-q_1) + N_2 P_2 (1+q_2)/(1-q_2) then
                     AED := N_1 P_1 (1+q_1)/(1-q_1) + N_2 P_2 (1+q_2)/(1-q_2), B_1 := N_1;
             end;
Loop 2:      while N_1 <= N_2 (1+q_2)/(1-q_2) (1-q_1)/(1+q_1) do begin
                 N_1 := N_1 + 1, N_2 := N_2 - 1:
                 P_1 := P_1 + p_{N_1}, P_2 := P_2 - p_{N_1};
                 if AED > N_1 P_1 (1+q_1)/(1-q_1) + N_2 P_2 (1+q_2)/(1-q_2) then
                     AED := N_1 P_1 (1+q_1)/(1-q_1) + N_2 P_2 (1+q_2)/(1-q_2), B_1 := N_1;
             end
             return (AED, B_1)
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Figure 1: The data allocation algorithm for unit length items and two channels with error probabilities q_{min} and q_{max}.

In Figure 1, an algorithm is shown which finds an optimal solution for the data allocation problem in the presence of unrecoverable channel transmission errors. In order to prove its correctness, note that the algorithm looks for a segmentation of minimum cost among all the admissible segmentations. Precisely, observed that N_1 coincides with the segmentation border B_1, the algorithm moves B_1 from left to right, one position at a time. In the initialization, the AED relative to the segmentation $(B_1 = 1)$ is computed and the condition $N_1P_1 > N_2P_2$ is checked, according to Lemma 2, to decide in which order the channels have to be taken. Then, Loop 1 computes the AEDs of consecutive segmentations up to the leftmost segmentation $(B_1 = N_1)$ such that $N_1P_1 > N_2P_2$. According to Corollary 1, at most $\lceil \frac{N}{2} \rceil$ segmentations are examined in Loop 1, and for each of them but the last one the channel with the largest error probability is used first. In contrast, in Loop 2 the channel with the minimum error probability is used first. Indeed, once the condition $N_1P_1 > N_2P_2$ becomes true at the end of Loop 1, it will remain satisfied in Loop 2 as the segmentation border moves right. According to Corollary 1, the last admissible segmentation is $(B_1 = (N - B_1)\frac{1+q_{max}}{1-q_{max}}\frac{1-q_{min}}{1+q_{min}})$. Finally, the solution with the minimum AED among all the so scanned solutions is returned.

The algorithm in Figure 1 requires $O(N)$ time, assuming that the items are already sorted, and $O(N \log N)$ time otherwise. Note that such a time complexity is the same as that achieved, when $K = 2$, by the Dichotomic algorithm, which is the fastest optimal algorithm for error-free channels. However, in the presence of unrecoverable transmission errors, Dichotomic does not always find an optimal solution because there is no guarantee that an optimal solution exists which satisfies Lemma 2 of [5].

In the particular case that all the channels have the same probability to fail, that is, $q_1 = \cdots = q_K = q$, the data allocation problem can still be optimally solved in polynomial time. This derives from Lemma 1 above and from Lemma 2 of [5], which together prove optimality in the particular case of error-free channels, that is, when $q = 0$. Indeed, when $q > 0$, similar proofs hold once the cost $C_{i,j}$ of assigning consecutive items $d_i, \ldots, d_j$ to the same channel is defined as $C_{i,j} = \frac{j-i+1}{2} \frac{1+q}{1-q} \sum_{h=i}^{j} p_h$ because the objective function given in Equation 9 becomes AED $= \frac{1}{2} \frac{1+q}{1-q} \sum_{k=1}^{K} N_k P_k$. In words, such lemmas show that, whenever the items $d_1, d_2, \ldots, d_N$ are sorted by non-increasing popularities, there always exists an optimal solution which is a segmentation and which can be found by the Dichotomic algorithm.

Another particular case that can be optimally solved in polynomial time arises when all the channels, but one, have the same probability to fail, namely, $q_1 = \cdots = q_{K-1} = q$ and $q_K = q'$. Such a problem can be optimally solved by using dynamic programming as follows. Let $C_{i,j} = \frac{j-i+1}{2} \frac{1+q}{1-q} \sum_{h=i}^{j} p_h$ and $C'_{i,j} = \frac{j-i+1}{2} \frac{1+q'}{1-q'} \sum_{h=i}^{j} p_h$ be the cost of assigning consecutive items $d_i, \ldots, d_j$ to a channel with error probability q and q', respectively. Moreover, let $opt_{k,n}$ be the cost of an optimal segmentation for the first n items using k channels all having the same error probability q. Similarly, let $opt'_{k,n}$ be the cost of an optimal segmentation when one of the k channels has error probability q'. Clearly, $opt_{1,n} = C_{1,n}$ and $opt'_{1,n} = C'_{1,n}$. The optimal solution $opt'_{K,N}$ can be derived applying the following recurrence, which exploits the fact that there is exactly one channel with different error probability q':

$$opt'_{k,n} = \min_{1 \leq \ell \leq n-1} \{ \min \{ opt_{k-1,\ell} + C'_{\ell+1,n}, opt'_{k-1,\ell} + C_{\ell+1,n} \} \} \quad 1 < k \leq K \qquad (10)$$

where

$$opt_{k,n} = \min_{1 \leq \ell \leq n-1} \{ opt_{k-1,\ell} + C_{\ell+1,n} \} \quad 1 < k \leq K-1 \qquad (11)$$

Recurrence 11 computes the optimal AED when no channel with error probability q' is used, while Recurrence 10 finds the optimal AED when exactly one channel with error probability q' is used. Precisely, this last recurrence considers that the channel with error probability q' is either the k-th one or one out of the first $k-1$ channels. Overall, the time complexity is $O(N^2 K)$.

In the general case that the error probabilities of the K channels are not the same, the algorithm in Figure 1 could be generalized by considering the items sorted by non-increasing popularities and by generating all the possible $\binom{N}{K-1}$ segmentations. Hence, for each segmentation, the groups are reindexed in such a way that their $N_j P_j$ are non-increasing, and the so reindexed groups are assigned to the channels taken by non-decreasing error probabilities. Clearly, such an algorithm requires $O(N^{K-1} K \log K + N \log N)$ time, which is polynomial only when $K = O(1)$. In order to have a polynomial time, the solution optimality can be compromised. Therefore, both the Dichotomic and Dlinear algorithms can be modified to handle such a general case with no guarantee that the so found solutions are optimal. Indeed, one can only show that the above mentioned lemmas hold true for a fixed ordering of the channels. A reasonable greedy criterium can be that of assigning the most popular items to the most reliable channels, that is, indexing the channels so that $q_1 \leq q_2 \leq \cdots \leq q_K$. It is easy to see from Lemma 2 that such a choice gives the optimum when there are exactly K items. Hence, Recurrences 5 and 6 can be adapted to the objective function given in Equation 9 by properly redefining the $C_{i,j}$ costs so that they depend on the channels too. Thus, letting the cost $C_{i,j;k}$ of assigning consecutive items $d_i, \ldots, d_j$ to channel k be $C_{i,j;k} = \frac{j-i+1}{2} \frac{1+q_k}{1-q_k} \sum_{h=i}^{j} p_h$, the new recurrences for the Dichotomic and Dlinear algorithms are derived from the old ones by replacing $C_{i,j}$ with $C_{i,j;k}$. All the $C_{i,j;k}$'s can be calculated in $O(NK)$ time via proper prefix-sum computations, assuming that the items are already sorted, and thus the time complexities of the Dichotomic and Dlinear algorithms remain the same.

3.2 Non-unit length items

Consider now items with non-unit length and recall that Z_k is the period of channel k. In order to receive an item d_i of length z_i over channel k, a client has to listen for z_i consecutive error-free packet transmissions, which happens with probability $(1-q_k)^{z_i}$. Hence, the error probability for item d_i on channel k is $Q_{z_i,k} = 1 - (1-q_k)^{z_i}$.

In the case that the first transmission of d_i heard by the client is error-free, the client has to wait on the average $\frac{Z_k}{2}$ time units with probability $1 - Q_{z_i,k}$. Instead, the client waits on the average for $\frac{Z_k}{2} + Z_k$ time units with probability $Q_{z_i,k}(1 - Q_{z_i,k})$ in the case that the first transmission of d_i is erroneous and the second one is error-free. In general, h bad transmissions of d_i followed by a good one lead to a delay of $\frac{Z_k}{2} + hZ_k$ time units with probability $Q_{z_i,k}^h (1 - Q_{z_i,k})$. Therefore, the expected delay becomes

$$t_i = \sum_{h=0}^{\infty} \left(\frac{Z_k}{2} + hZ_k \right) Q_{z_i,k}^h (1 - Q_{z_i,k}) = \frac{Z_k}{2} \frac{1 + Q_{z_i,k}}{1 - Q_{z_i,k}} \tag{12}$$

and Equation 1 can be rewritten as

$$\text{AED} = \sum_{i=1}^{N} t_i p_i = \frac{1}{2} \sum_{k=1}^{K} \left(Z_k \sum_{d_i \in G_k} \frac{1 + Q_{z_i,k}}{1 - Q_{z_i,k}} p_i \right) \tag{13}$$

Recalling that the items are indexed by non-increasing $\frac{p_i}{z_i}$ ratios, the new recurrences for the Dichotomic and Dlinear algorithms are derived from Recurrences 5 and 6, respectively, once the channels are indexed so that $q_1 \leq q_2 \leq \cdots \leq q_K$ and each $C_{i,j}$ is replaced by $C_{i,j;k} = \frac{1}{2}\left(\sum_{h=i}^{j} z_h\right)\left(\sum_{h=i}^{j} \frac{1+Q_{z_h,k}}{1-Q_{z_h,k}} p_h\right)$. All the $C_{i,j;k}$'s can be computed in $O(KH)$ time via prefix-sums, where $H = \min\{N \log z, z\}$ and $z = \max_{1 \leq h \leq N} z_h$. Therefore, the time complexities of the Dichotomic and Dlinear algorithms become, respectively, $O(K(H + N \log N))$ and $O(KH + KN + N \log N)$. Note that in such a case optimality is not guaranteed since the problem is computationally intractable already for error-free channels.

However, when there are only two channels having the same error probability $q = q_1 = q_2$, an optimal solution can be found applying the Knapsack algorithm simply replacing each popularity p_i with $p'_i = \frac{1+Q_{z_i}}{1-Q_{z_i}} p_i$ in Recurrence 7, and then finally selecting the entry $M_{N,\bar{j}}$ which minimizes $\frac{1}{2}\left(\bar{j} M_{N,\bar{j}} + (Z - \bar{j})(P' - M_{N,\bar{j}})\right)$, where $P' = \sum_{i=1}^{N} p'_i$ and $Q_{z_i} = 1-(1-q)^{z_i}$, for $1 \leq i \leq N$. The overall time complexity remains $O(NZ)$.

3.3 Simulation experiments

In this subsection, the behavior of the Dichotomic and Dlinear heuristics is tested in the case of Bernoulli channel error model. The heuristics were written in C++ and the experiments were run on an AMD Athlon X2 4800+ with 2 GB RAM. The above algorithms have been experimentally tested on benchmarks where the item popularities follow a Zipf distribution. Specifically, given the number N of items and a real number $0 \leq \theta \leq 1$, the item popularities are defined as

$$p_i = \frac{(1/i)^\theta}{\sum_{h=1}^{N}(1/h)^\theta} \qquad 1 \leq i \leq N$$

Note that the item popularities are already sorted in non-increasing order. In the above formula, θ is the *skew* parameter. In particular, $\theta = 0$ stands for a uniform distribution with $p_i = \frac{1}{N}$, while a higher θ implies a higher skew, namely the difference among the p_i values becomes larger. In the experiments, θ is chosen to be 0.8, as suggested in [22], while either N is set to 2500 and K varies in the range $10 \leq K \leq 500$, or K is set to 50 and N varies in the range $500 \leq N \leq 2500$. The channel error probabilities can assume the values 0.001, 0.01 and 0.1.

Figure 2 exhibits the AED obtained in the case that the data lengths are unitary and the error probabilities are not identical for all channels. In particular, the channels are partitioned into three equally-sized groups with error probability q, $2q$, and $3q$, respectively. In other words, $q_1 = \cdots = q_{\lfloor \frac{K}{3} \rfloor} = q$, $q_{\lfloor \frac{K}{3} \rfloor + 1} = \cdots = q_{\lfloor \frac{2}{3}K \rfloor} = 2q$, and $q_{\lfloor \frac{2}{3}K \rfloor + 1} = \cdots = q_K = 3q$. One can observe that, when $q = 0.001$ and 0.01, the reported AEDs almost coincide with those where the channels are error-free. In other words, such small error probability values scarcely affect the average expected delay, which remains the optimal one found by

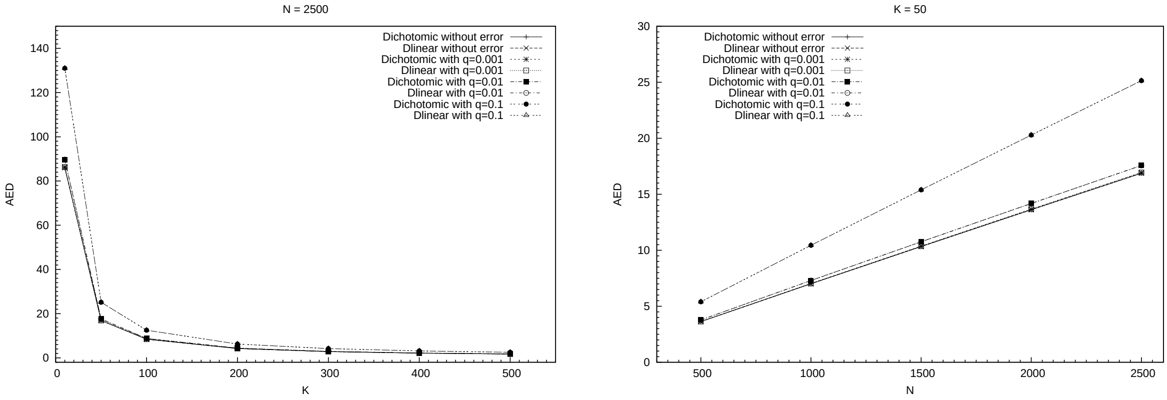

Figure 2: *Results for unit lengths when the channels are partitioned into three groups of the same size with error probability q, $2q$, and $3q$, respectively.*

the Dichotomic algorithm in the case of channels with no error. Whereas, the larger value $q = 0.1$ worsens the AED when the number K of channels is small with respect to the number N of items. Noting that all the channels have at least an error probability of $q = 0.1$, the AED in presence of errors must be at least $\frac{1+q}{1-q} = 1.22$ times the AED without errors. This is consistent with the AED reported in Figure 2, which is about 1.44 times the AED without errors, as computed by both the Dlinear and Dichotomic algorithms.

Consider now data items whose lengths are non-unitary. In the experiments, the item lengths z_i are integers randomly generated according to a uniform distribution in the range $1 \leq z_i \leq 10$, for $1 \leq i \leq N$. In addition, the reported results are averaged over 3 independent experiments. Moreover, since the data allocation problem is computationally intractable when data lengths are non-unitary, lower bounds for a non-unit length instance are derived by transforming it into a unit length instance as follows. Each item d_i of popularity p_i and length z_i is decomposed into z_i items of popularity $\frac{p_i}{z_i}$ and length 1. Since more freedom has been introduced, it is clear that the optimal AED for the so transformed problem is a lower bound on the AED of the original problem. Since the transformed problem has unit lengths, when all the channels are either error-free or have the same error probability, the optimal AED can be obtained by running the polynomial time Dichotomic algorithm.

Figure 3 shows the experimental results for non-unit lengths in the case that the error probability q is 0.01 for all channels. One can note that the two above mentioned lower bounds almost coincide. Indeed, the AED of the transformed unit length instance in the presence of errors is $\frac{1+q}{1-q} = 1.02$ times the AED of the same transformed instance without errors. One can also note that, since the average data item length is 5, the AED of the original instance in the presence of errors should be about $\frac{1+Q}{1-Q} = 1.10$ times the AED of the same original instance in the absence of errors, where $Q = 1 - (1 - 0.01)^5 = 0.05$. This can be easily checked in the graphic, e.g., for $K = 10$, where the ratio between the two AEDs is

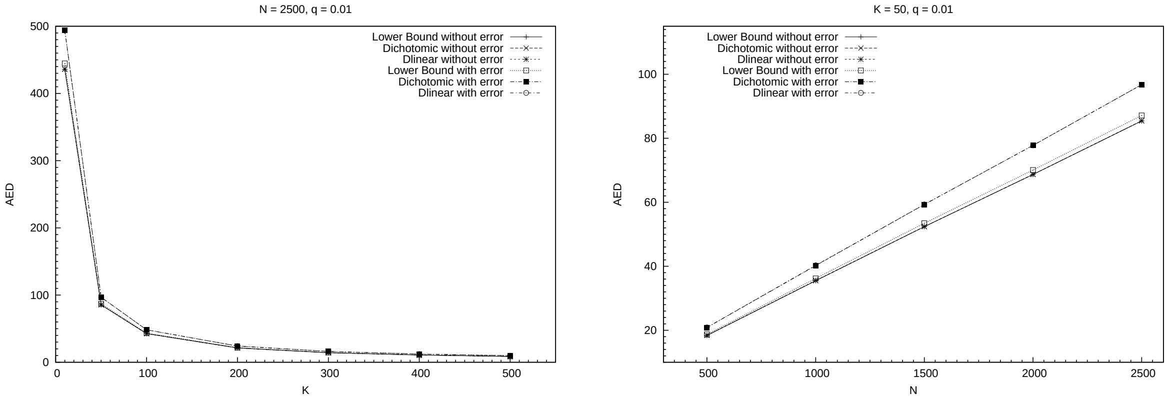

Figure 3: *Results for non-unit lengths when all the channels have the same error probability* $q = 0.01$.

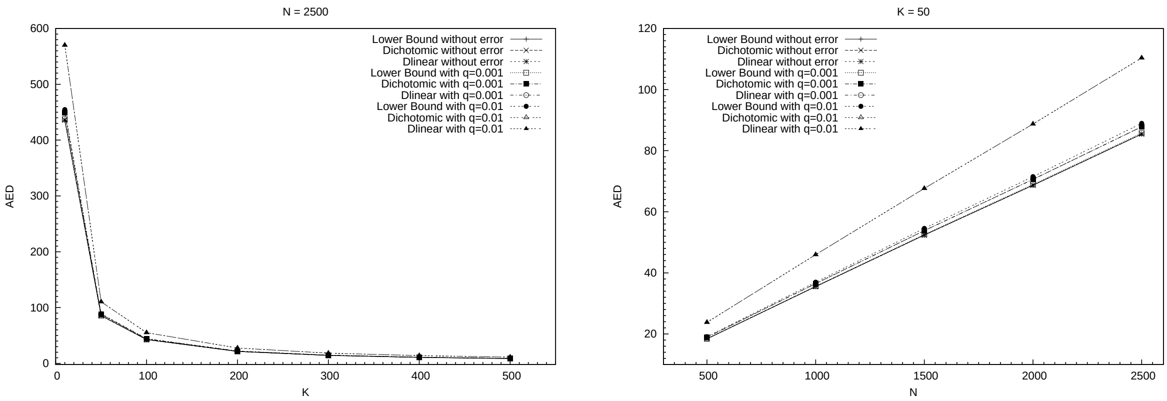

Figure 4: *Results for non-unit lengths when the channels are partitioned into three groups of the same size with error probability* q, $2q$, *and* $3q$, *respectively.*

about $\frac{500}{450} = 1.11$.

When the error probabilities of the channels are not identical, it is not known how to compute in polynomial time the optimal AED for the transformed unit length instance, which gives a lower bound to the original instance. Therefore, such an optimal AED is replaced in the experiments by the AED obtained running the Dichotomic algorithm on the transformed instance, which nonetheless remains a lower bound of the AED produced by the Dichotomic algorithm for the original non-unit instance.

Figures 4 and 5 plot the AEDs obtained for non-unit lengths and three equally-sized channel groups with error probability q, $2q$, and $3q$. When $q = 0.001$, the AEDs in Figure 4 almost coincide with those where the channels are error-free, as happened in the case of unit lengths. When $q = 0.01$, since the average data item length is 5 and the average channel error probability is 0.02, the AED of the original instance in the presence of error should be about $\frac{1+Q}{1-Q} = 1.22$ times the AED of the same original instance in the absence of error, where

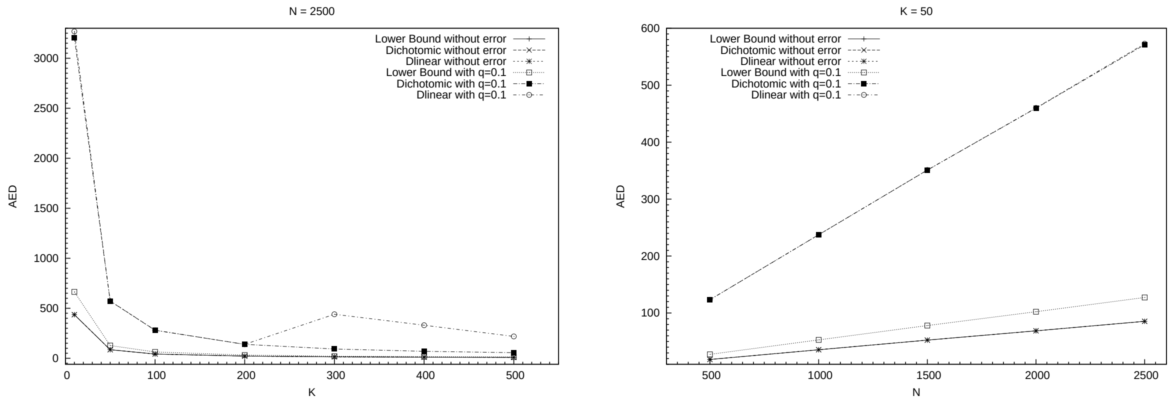

Figure 5: *Results for non-unit lengths when the channels are partitioned into three groups of the same size with error probability* 0.1*,* 0.2*, and* 0.3*, respectively.*

$Q = 1 - (1 - 0.02)^5 = 0.10$. In Figure 4, the largest ratios between the two above mentioned AEDs occur for small values of K, e.g., when $K = 10$ such a ratio is about $\frac{570}{440} = 1.29$. When $q = 0.1$, a similar reasoning leads to $Q = 1 - (1 - 0.2)^5 = 0.68$ and $\frac{1+Q}{1-Q} = 5.25$, while the largest ratio, for $K = 10$, is about $\frac{3200}{450} = 7.11$, as one can see in Figure 5. Moreover, one notes that the Dlinear algorithm, which searches in a smaller solution space than that of the Dichotomic, behaves worse for large values of K.

4 Gilbert-Elliot channel error model

In this section, the channel error behavior is assumed to follow a simplified Gilbert-Elliot model, which is a two-state time-homogeneous discrete time Markov chain [20]. At each time instant, a channel can be in one of two states. The state 0 denotes the *good* state, where the channel works properly and thus a packet is received with no errors. Instead, the state 1 denotes the *bad* state, where the channel is subject to failure and hence a packet is received with an unrecoverable error. Let $X_0, X_1, X_2, \ldots$ be the states of the channel at times $0, 1, 2, \ldots$. The time between X_u and X_{u+1} corresponds to the length of one packet. The initial state X_0 is selected randomly. As depicted in Figure 6, the probability of transition from the good state to the bad one is denoted by b, while that from the bad state to the good one is g. Hence, $1 - b$ and $1 - g$ are the probabilities of remaining in the same state, namely, in the good and bad state, respectively. Formally, $Prob[X_{u+1} = 0 | X_u = 0] = 1 - b$, $Prob[X_{u+1} = 0 | X_u = 1] = g$, $Prob[X_{u+1} = 1 | X_u = 1] = 1 - g$, and $Prob[X_{u+1} = 1 | X_u = 0] = b$.

It is well known that the *steady-state* probability of being in the good state is $P_G = \frac{g}{b+g}$, while that of being in the bad state is $P_B = \frac{b}{b+g}$. This Markovian process has mean $\mu = P_B$,

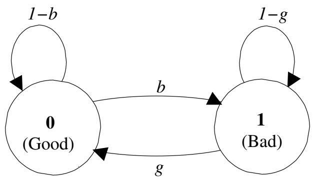

Figure 6: *The Gilbert-Elliot channel error model.*

variance $\sigma^2 = \mu(1-\mu) = \frac{bg}{(b+g)^2}$, and autocorrelation function $r(\nu) = P_B + (1-P_B)(1-b-g)^\nu$, where $b+g < 1$ is assumed. Since the system is memoryless, the state holding times are geometrically distributed. The mean state holding times for the good state and the bad state are, respectively, $\frac{1}{b}$ and $\frac{1}{g}$. This means that the channel exhibits error bursts of consecutive ones whose mean length is $\frac{1}{g}$, that are separated by gaps of consecutive zeros whose mean length is $\frac{1}{b}$.

4.1 Unit length items

Assume that the item lengths are unitary, i.e., $z_i = 1$ for $1 \leq i \leq N$. Recall that in such a case the period of channel k is N_k. If a client waits for item d_i on channel k, and no error occurs in the first transmission of d_i, then the client waits on the average $\frac{N_k}{2}$ time units with probability $P_G = 1 - P_B$. Instead, if an error occurs during the first transmission of d_i and there is no error in the second transmission, then the average delay experienced by the client is $\frac{N_k}{2} + N_k$ time units with probability $P_B(1-r(N_k))$. In general, when there are h erroneous transmissions of d_i followed by an error-free one, the client average delay is $\frac{N_k}{2} + hN_k$ time units with probability $P_B(r(N_k))^{h-1}(1-r(N_k))$. Thus, the expected delay is equal to

$$\frac{N_k}{2}P_G + P_B(1-r(N_k))\sum_{h=1}^{\infty}(\frac{N_k}{2} + hN_k)(r(N_k))^{h-1} = \frac{N_k}{2}P_G + P_B\frac{N_k}{2} + P_B\frac{N_k}{1-r(N_k)}$$

because $\sum_{h=1}^{\infty}(r(N_k))^{h-1} = \frac{1}{1-r(N_k)}$ and $\sum_{h=1}^{\infty} h(r(N_k))^{h-1} = \frac{1}{(1-r(N_k))^2}$. Hence, the expected delay t_i and the objective function become, respectively

$$t_i = \frac{N_k}{2}\left(1 + \frac{2P_B}{1-r(N_k)}\right) \tag{14}$$

$$\text{AED} = \sum_{i=1}^{N} t_i p_i = \frac{1}{2}\sum_{k=1}^{K}\left(N_k\left(1 + \frac{2P_B}{1-r(N_k)}\right)\sum_{d_i \in G_k} p_i\right) \tag{15}$$

The following result shows that there is an optimal solution where the items are sorted by non-increasing popularities.

Lemma 3. *Let G_h and G_j be two groups in an optimal solution. Let d_i and d_k be items with $d_i \in G_h$ and $d_k \in G_j$. If $N_h\left(1+\frac{2P_B}{1-r(N_h)}\right) < N_j\left(1+\frac{2P_B}{1-r(N_j)}\right)$, then $p_i \geq p_k$. Similarly, if $p_i > p_k$, then $N_h\left(1+\frac{2P_B}{1-r(N_h)}\right) \leq N_j\left(1+\frac{2P_B}{1-r(N_j)}\right)$.*

Proof. By contradiction, let $G_1, G_2, \ldots, G_K$ be an optimal solution for which there exist G_h and G_j such that $N_h\left(1+\frac{2P_B}{1-r(N_h)}\right) < N_j\left(1+\frac{2P_B}{1-r(N_j)}\right)$ and $p_i < p_k$. Consider now another solution obtained by exchanging d_i with d_k in the two groups G_h and G_j. The difference in the AED of the two solutions is $\left(N_h\left(1+\frac{2P_B}{1-r(N_h)}\right) - N_j\left(1+\frac{2P_B}{1-r(N_j)}\right)\right)(p_i - p_k) > 0$, because $p_i - p_k < 0$ and $N_h\left(1+\frac{2P_B}{1-r(N_h)}\right) < N_j\left(1+\frac{2P_B}{1-r(N_j)}\right)$. Hence, a better solution is achieved contradicting the optimality assumption. The last part of the lemma is proved similarly. □

The above lemma implies that there is an optimal solution which is a segmentation. Such a solution can be found in $O(N^2K)$ time by the DP algorithm, whose new recurrence is derived from Recurrence 4 by setting $C_{i,j} = \frac{j-i+1}{2}\left(1+\frac{2P_B}{1-r(j-i+1)}\right)\sum_{h=i}^{j} p_h$.

In the general case where the steady-state probabilities of being in the bad state are not identical for all channels, Equations 14 and 15 can be easily generalized. Then, both the Dichotomic and Dlinear algorithms can still be applied to find sub-optimal solutions, after indexing the channels by non decreasing P_B's, namely $P_{B_1} \leq \cdots \leq P_{B_K}$, and replacing $C_{i,j}$ with

$$C_{i,j;k} = \frac{j-i+1}{2}\left(1+\frac{2P_{B_k}}{1-r_k(j-i+1)}\right)\sum_{h=i}^{j} p_h$$

where $r_k(\nu) = P_{B_k} + (1-P_{B_k})(1-b_k-g_k)^\nu$. As usual, all the $C_{i,j;k}$'s can be computed in $O(NK)$ time via prefix-sums.

In the special case where there are only two channels, an optimal solution can be efficiently found by exploiting the properties of the AED objective function. Indeed, the problem is to find a partition G_1 and G_2 such that $\frac{1}{2}\left(N_1\left(1+\frac{2P_{B_1}}{1-r_1(N_1)}\right)P_1 + N_2\left(1+\frac{2P_{B_2}}{1-r_2(N_2)}\right)P_2\right)$ is minimized. Since $P_2 = 1-P_1$ and $N_2 = N - N_1$, the AED above can be rewritten as:
$\frac{1}{2}\left(P_1\left(N_1\left(2+\frac{2P_{B_1}}{1-r_1(N_1)}+\frac{2P_{B_2}}{1-r_2(N-N_1)}\right) - N\left(1+\frac{2P_{B_2}}{1-r_2(N-N_1)}\right)\right) + \left(1+\frac{2P_{B_2}}{1-r_2(N-N_1)}\right)(N-N_1)\right)$
When N_1 is fixed to a particular value, the AED is minimized by assigning to group G_1 the N_1 items with either the smallest or largest popularities, depending on whether $\alpha = N_1\left(2+\frac{2P_{B_1}}{1-r_1(N_1)}+\frac{2P_{B_2}}{1-r_2(N-N_1)}\right) - N\left(1+\frac{2P_{B_2}}{1-r_2(N-N_1)}\right)$ is positive or not, respectively. Such a property implies that there is an optimal solution which is a segmentation and which can be found by scanning all the possible values of N_1 once the items have been sorted by non-increasing popularities. The resulting algorithm, shown in Figure 7, has an $O(N)$ running time, provided that the items are sorted. The algorithm starts computing in $O(N)$ time all the autocorrelation values, $r_1(i)$ and $r_2(i)$, as well as the popularity prefix sums, $\Pi_i = \sum_{j=1}^{i} p_i$, for $1 \leq i \leq N$. Then, for each i, with $1 \leq i \leq N-1$, it assigns to G_1 either

Input:	N items sorted by non-increasing popularities $\{p_1, p_2, \ldots, p_N\}$;
	$K = 2$ channels; steady-state probabilities P_{B_1} and P_{B_2};
Initialize:	$\Pi_i := \sum_{j=1}^{i} p_i$, for $1 \le i \le N$;
	$r_k(i) := P_{B_k} + (1 - P_{B_k})(1 - b_k - g_k)^i$, for $k = 1, 2$ and $1 \le i \le N$;
	AED $:= +\infty$;
Loop 1:	**for** $i := 1$ **to** $N - 1$ **do begin**
	$\alpha := i\left(2 + \frac{2P_{B_1}}{1-r_1(i)} + \frac{2P_{B_2}}{1-r_2(N-i)}\right) - N\left(1 + \frac{2P_{B_2}}{1-r_2(N-i)}\right)$;
	if $\alpha > 0$ **then** $P_1 := \Pi_N - \Pi_{N-i+1}$, $\beta := N - i$
	else $P_1 := \Pi_i$, $\beta := i$;
	if AED $> \frac{1}{2}\left(\alpha P_1 + \left(1 + \frac{2P_{B_2}}{1-r_2(N-i)}\right)(N-i)\right)$ **then**
	AED $:= \frac{1}{2}\left(\alpha P_1 + \left(1 + \frac{2P_{B_2}}{1-r_2(N-i)}\right)(N-i)\right)$, $B_1 := \beta$;
	end
	return (AED, B_1)

Figure 7: The data allocation algorithm for unit length items and two channels with steady-state probabilities P_{B_1} and P_{B_2}.

the last i items, if $\alpha > 0$, or the first i items, if $\alpha \le 0$. Finally, it returns the solution with minimum AED among all the $N - 1$ scanned solutions.

4.2 Non-unit length items

Let us now deal with items having non-unit lengths. Recall that Z_k is the period of channel k and that a client has to listen for z_i consecutive error-free packet transmissions in order to receive the item d_i over channel k.

Consider now the first transmission of item d_i heard by a client. Let $\hat{P}_B(s)$ denote the probability that in such a transmission the s-th packet is the first erroneous packet, where $1 \le s \le z_i$. Formally,

$$\hat{P}_B(s) = \begin{cases} P_B & \text{if } s = 1 \\ (1 - P_B)(1 - b)^{s-2} b & \text{if } 2 \le s \le z_i \end{cases}$$

Consider now two consecutive transmissions of item d_i heard by a client, the first of which is erroneous. Let $\bar{P}_B(s, \sigma)$ denote the probability that, in the second transmission, the first erroneous packet is the s-th one given that in the previous transmission the first erroneous packet was the σ-th one. Thus,

$$\bar{P}_B(s, \sigma) = \begin{cases} r(Z_k + 1 - \sigma) & \text{if } s = 1 \\ (1 - r(Z_k + 1 - \sigma))(1 - b)^{s-2} b & \text{if } 2 \le s \le z_i \end{cases}$$

Finally, let $\bar{P}_G(\sigma)$ denote the probability that a whole transmission of d_i is error-free given that in the previous transmission of d_i the first erroneous packet was the σ-th one:

$$\bar{P}_G(\sigma) = (1 - r(Z_k + 1 - \sigma))(1-b)^{z_i - 1}$$

Note that all the $\hat{P}_B(s)$ and $\bar{P}_B(s, \sigma)$'s can be computed in pseudo-polynomial time, that is in a time polynomial in the parameters Z and z, where $Z = \sum_{i=1}^{N} z_i$ and $z = \max_{1 \le i \le N} z_i$.

To evaluate the expected delay t_i, observe that if the first transmission of d_i heard by the client is error-free, the client has to wait on the average $\frac{Z_k}{2}$ time units with probability $(1 - P_B)(1-b)^{z_i - 1}$. Instead, the client waits on the average for $\frac{Z_k}{2} + Z_k$ time units with probability $\sum_{s_0=1}^{z_i} \hat{P}_B(s_0)\bar{P}_G(s_0)$ in the case that the first transmission of d_i is erroneous and the second one is error-free. Moreover, two bad transmissions of d_i followed by a good one lead to a delay of $\frac{Z_k}{2} + 2Z_k$ time units with probability $\sum_{s_0=1}^{z_i} \left[\hat{P}_B(s_0) \sum_{s_1=1}^{z_i} \bar{P}_B(s_1, s_0)\bar{P}_G(s_1)\right]$. Thus, in general, the expected delay is $t_i = \frac{Z_k}{2}(1 - P_B)(1-b)^{z_i-1} + \sum_{h=1}^{\infty} \left[\left(\frac{Z_k}{2} + hZ_k\right) \sum_{s_0=1}^{z_i} \left[\hat{P}_B(s_0) \sum_{s_1=1}^{z_i} \left[\bar{P}_B(s_1, s_0) \sum_{s_2=1}^{z_i} \left[\bar{P}_B(s_2, s_1) \cdots \sum_{s_{h-1}=1}^{z_i} \left[\bar{P}_B(s_{h-1}, s_{h-2})\bar{P}_G(s_{h-1})\right] \cdots\right]\right]\right]\right]$.

Since finding a closed formula for t_i seems to be difficult, an approximation t_i^m of the expected delay can be computed by truncating the above series at the m-th term, for a given constant value m. Indeed, experimental tests show that the series converges already for small values of m, as it will be checked in Subsection 4.3. Thus, the average expected delay becomes AED $= \sum_{i=1}^{N} t_i^m p_i$. Recalling that the items are indexed by non-increasing $\frac{p_i}{z_i}$ ratios, the Dichotomic and Dlinear algorithms can be applied once each $C_{i,j}$ is computed as $\sum_{h=i}^{j} t_h^m p_h$. Fixed i and j, the time for computing $C_{i,j}$ is derived as follows. Assuming a proper prefix-sum has been done as a preprocessing, $Z_k = \sum_{h=i}^{j} z_h$ can be retrieved in $O(1)$ time, while the computation of t_h^m requires $O(z_h^m)$ time. Therefore, in the worst case, the computation of $C_{i,j}$ takes $O(Nz^m)$ time, and that of all the $C_{i,j}$'s costs $O(N^3 z^m)$ time, which is pseudo-polynomial. Hence, the time for computing the $\hat{P}_B(s)$'s, $\bar{P}_B(s, \sigma)$'s, and $C_{i,j}$'s leads to a pseudo-polynomial time complexity for both the Dichotomic and Dlinear algorithms.

As in the unit length case, if the steady-state probabilities of being in the bad state are not identical for all channels, then Dichotomic and Dlinear can be run after the channels are indexed so that $P_{B_1} \le \cdots \le P_{B_K}$ and each $C_{i,j}$ is replaced with $C_{i,j;k} = \sum_{h=i}^{j} t_h^m(k) p_h$, where $t_h^m(k)$ is computed as t_h^m by substituting P_{B_k} for P_B. Clearly, the computation of all the $C_{i,j;k}$'s takes $O(N^3 K z^m)$ time.

4.3 Simulation experiments

This subsection presents the experimental tests for the Dichotomic and Dlinear heuristics in the case of the Gilbert-Elliot channel error model. In the experiments for items of unit length, the item popularities follow a Zipf distribution with $\theta = 0.8$, while either $N = 2500$

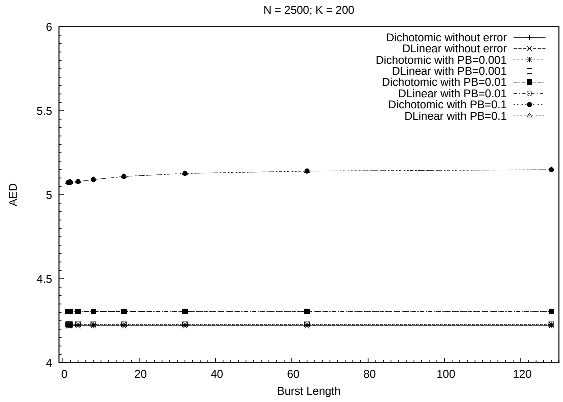

Figure 8: *The AED behavior versus the mean error burst length.*

and $10 \leq K \leq 500$, or $K = 50$ and $500 \leq N \leq 2500$, as in Subsection 3.3. Moreover, the steady-state probability P_B of being in the bad state can assume the values 0.001, 0.01 and 0.1, while the mean error burst length $\frac{1}{g}$ is fixed to 10. Note that b is derived as $g\frac{P_B}{1-P_B}$ once P_B and $\frac{1}{g}$ are fixed. However, the choice of $\frac{1}{g}$ is not critical because the sensitivity of the AED to $\frac{1}{g}$ is low, as depicted in Figure 8, for $1 < \frac{1}{g} \leq 130$. Note that the choice of such an upper bound on $\frac{1}{g}$ is not restrictive because the probability of having a burst with length n is $g(1-g)^{n-1}$, which is negligible as n grows.

Figure 9 exhibits the AED obtained in the case where the data lengths are unitary and the steady-state probabilities are not identical for all channels. As in the Bernoulli error model, the channels are indexed in such a way that $P_{B_1} = \cdots = P_{B_{\lfloor \frac{K}{3} \rfloor}} = P_B$, $P_{B_{\lfloor \frac{K}{3} \rfloor+1}} = \cdots = P_{B_{\lfloor \frac{2}{3}K \rfloor}} = 2P_B$, and $P_{B_{\lfloor \frac{2}{3}K \rfloor+1}} = \cdots = P_{B_K} = 3P_B$. One can observe that, when $P_B = 0.001$ and 0.01, the reported AEDs almost coincide with those where the channels are error-free, whereas the AED worsens when $P_B = 0.1$. Noting that in this latter case the steady-state probability is 0.2 on the average, and thus $1 + \frac{2P_B}{1-r(N_k)} \simeq 1.40$ in Equation 14, one expects that the AED in the presence of errors should be about 40% larger than that in the absence of errors. This is confirmed by the results reported in Figure 9, where the experimental AED is about 44% larger than in the error-free case.

Consider now data items whose lengths are non-unitary. Since the algorithms take pseudo-polynomial time, a restricted set of experiments is performed. In the experiments, the number K of channels is set to 50, the number N of items varies between 500 and 2000, the item popularities follow a Zipf distribution with $\theta = 0.8$, and the item lengths z_i are integers randomly generated according to a uniform distribution in the range $1 \leq z_i \leq 10$, for $1 \leq i \leq N$. All the K channels have the same staedy-state probability P_B, which assumes the values 0.001, 0.01, and 0.1. The reported results are averaged over 3 independent

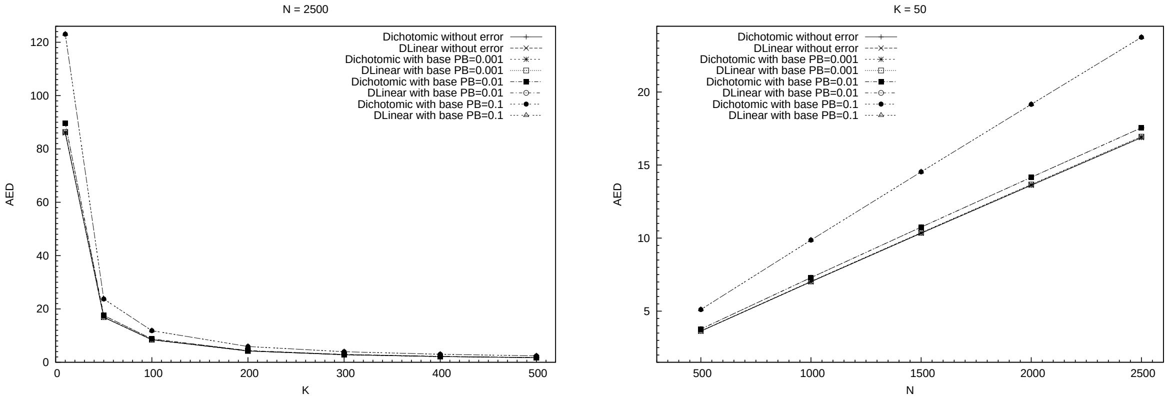

Figure 9: *Results for unit lengths when the channels are partitioned into three groups of the same size with steady-state probability P_B, $2P_B$, and $3P_B$, respectively.*

m	t_i^m
1	25.9150699
2	25.9382262
3	25.9388013
4	25.9388156
5	25.9388160
6	25.9388167

(a)

m	t_i^m
1	25.1989377
2	25.2537833
3	25.2689036
4	25.2730723
5	25.2745215
6	25.2745384

(b)

Table 1: *Values of t_i^m when: (a) $z_i = 10$, $Z_k = 50$, $\frac{1}{g} = 10$, and $P_B = 0.01$; and (b) $z_i = 5$, $Z_k = 50$, $\frac{1}{g} = 10$, and $P_B = 0.16$.*

experiments. The expected delay of item d_i is evaluated by computing t_i^5, that is truncating at the fifth term the series giving t_i. Indeed, as shown in Table 1 for $z_i = 10$, $Z_k = 50$, $\frac{1}{g} = 10$, and $P_B = 0.01$ and for $z_i = 5$, $Z_k = 50$, $\frac{1}{g} = 10$, and $P_B = 0.1$, at the fifth term the series giving t_i is already stabilized up to the fourth decimal digit.

Since the data allocation problem is computationally intractable when data lengths are non-unit, lower bounds for non-unit length instances are derived by transforming them into unit length instances, as explained in Subsection 3.3. Moreover, since the steady-state probability P_B is the same for all channels, the AEDs giving the lower bounds are obtained by running the DP algorithm as explained in Subsection 4.1.

Figure 10 shows the experimental results for non-unit lengths, where P_B assumes the values 0.001, 0.01 and 0.1. In this figure, lower bounds are shown for both error-free and error-prone channels. One notes that, for every value of P_B, the behavior of both the Dichotomic and Dlinear algorithms is identical. When $P_B = 0.001$, both algorithms provide optimal solutions because their AEDs almost coincide with the lower bound for channels

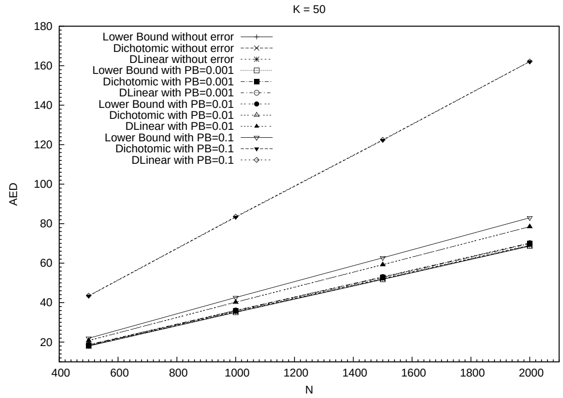

Figure 10: *Results for non-unit lengths when all the channels have the same steady-state probability* P_B*, which assumes the values* 0.001*,* 0.01*, and* 0.1*.*

without errors. When $P_B = 0.01$, the AEDs of both the Dichotomic and Dlinear algorithms are 12% larger than the lower bound in the presence of errors. In the last case, namely $P_B = 0.1$, the AEDs found by the algorithms are as large as twice those of the lower bound in presence of errors. However, such a value of P_B represents an extremal case which should not arise in practice (e.g. see [12]).

5 Conclusions

This paper studied the problem of allocating N data to K channels, assuming flat data scheduling per channel and the presence of unrecoverable channel transmission errors. The objective was that of minimizing the average expected delay experienced by clients. The behavior of two polynomial time heuristics has been experimentally tested modelling the channel error by means of the Bernoulli model as well as the Gilbert-Elliot one. Such heuristics were derived by properly redefining the recurrences in the dynamic programming algorithms previously presented for error-free channels. Extensive simulations showed that such heuristics provide good sub-optimal solutions when tested on benchmarks whose item popularities follow Zipf distributions. In particular, for small channel error probabilities, the average expected delay is almost the same as the optimal one found in the case of channel without errors. However, some subcases have been detected where an optimal solution can be found in polynomial or pseudo-polynomial time. All the complexity results are summarized in Table 2, where Z is the sum of the item lengths. The first row of the table shows the results previously known in the literature in the case of error-free channels [5]. All the new results proved in the present paper in the case of channels subject to transmission errors are exhibited in the remaining rows. Observe in the table that, since the problem is already

		Unit lengths		Non-unit lengths	
		$K = 2$	$K > 2$	$K = 2$	$K > 2$
Error-free		$O(N \log N)$	$O(NK \log N)$	$O(NZ)$	Strong NP-hard
Bernoulli	$q_1 = \cdots = q_K$	$O(N \log N)$	$O(NK \log N)$	$O(NZ)$	Strong NP-hard
	$q_1 \neq \cdots \neq q_K$	$O(N \log N)$	open	NP-hard	Strong NP-hard
Gilbert-Elliot	$P_{B_1} = \cdots = P_{B_K}$	$O(N \log N)$	$O(N^2K)$	NP-hard	Strong NP-hard
	$P_{B_1} \neq \cdots \neq P_{B_K}$	$O(N \log N)$	open	NP-hard	Strong NP-hard

Table 2: *Complexity results for optimal data allocation on multiple channels.*

computationally intractable for non-unit lengths and error-free channels, its computational complexity in the presence of errors remains an open issue only for the cases involving items with unit lengths and channels with different error probabilities. Nonetheless, experiments showed that near optimal solutions are found by the heuristics also in these cases. Finally, as regard to the non-unit length case, an interesting open question is that of determining whether a closed formula for computing the item expected delays exists or not when the Gilbert-Elliot model is adopted.

Acknowledgement

This work has been supported by ISTI-CNR under the BREW research grant. The C++ code used in the simulations was written by G. Spagnardi.

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