Verifying Data in Space and Time

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joint work mostly with Vincenzo Ciancia, Diego Latella and Michele Loreti

Consiglio Nazionale delle Ricerche
ISTITUTO DI SCIENZA E TECNOLOGIE DELL'INFORMAZIONE "A. FAEDO"

COOPERATION

DataMod 2019, 7-8 October 2019, Porto - Portugal


Space: the final frontier
GIF

Introduction

Origins of Spatial Reasoning

"Space, like time, is one of the most fundamental categories of human cognition.

It structures all our activities and relationships with the external world.

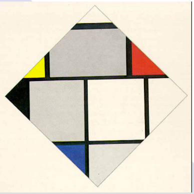
It also structures many of our reasoning capabilities: it serves as the basis for many metaphors, including temporal, and gave rise to mathematics itself, geometry being the first formal system known."

(Laure Vieu, 1997)

Physical Sciences

Pure Mathematics

Ordinary/
Partial
Differential
Equations

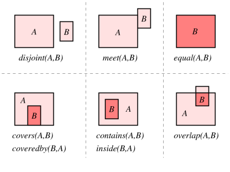


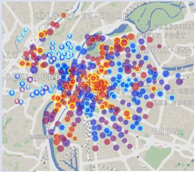
Topology
Modal Logics

Decidability
Satisfiability

SPACE

Region
Connection
Calculus





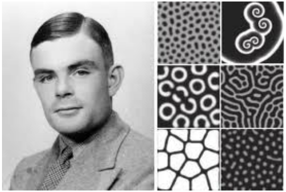
Model checking

Artificial Intelligence

Collective Adaptive Systems

Collective Adaptive Systems

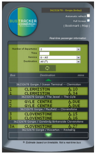
Examples of decentralised collective adaptive behaviour in nature:



Designing CAS for a smart society

The development of a formal verification framework for smart urban transport and smart grid.



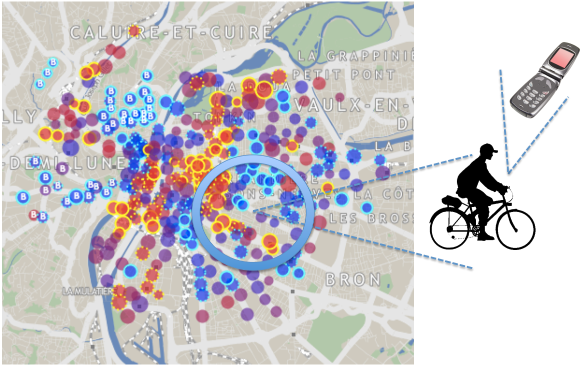
The long term objective is to support fair and efficient management of resources in large scale systems of heterogenous components that are spatially distributed and have possibly competing goals.

quanticol

blog.inf.ed.ac.uk/quanticol/

A Bike Sharing System

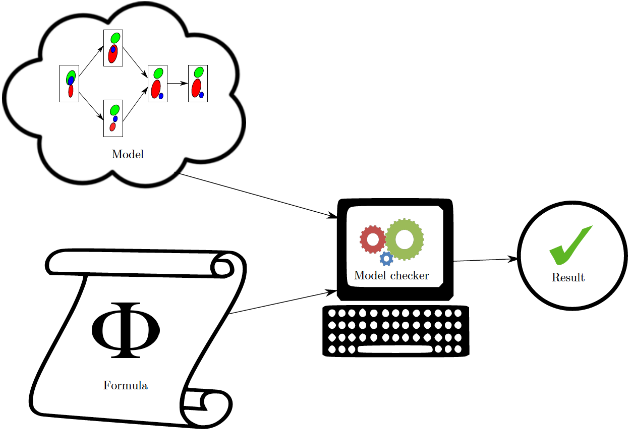
Continuous or discrete space? Space and time? Images? Points or sets?



Continuous space, discrete regular grid, graph of stations, street map

O'Brien's map of bike sharing `www.citylab.com`

Spatial-temporal Model Checking?



Unified Framework for Spatial Model Checking?

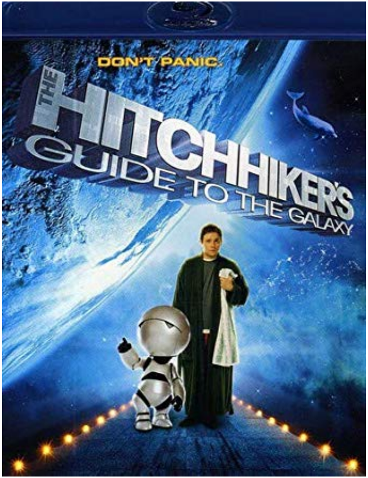
- Generalising some topological notions
- Bridging the gap between continuous and discrete space
- Spatial Logics for Model Checking

Bringing us to explore

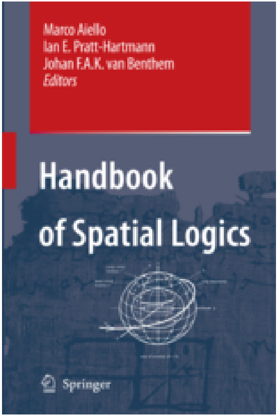
Closure Spaces and Quasi-discrete Closure Spaces

following up on work by, a.o., A. Galton and M. B. Smyth et al.

Hitchhikers Guide to the Galaxy



Handbook of Spatial Logics



Handbook of Spatial Logics
Aiello, Pratt-Hartmann and van Benthem (Eds.), Springer, 2007

PART I

Logics and Space

Topological Space

A pair (X, O) where

- $X \neq \emptyset$ is a set
- O is a collection of open sets $O \subseteq \mathcal{P}(X)$

such that

- $\emptyset, X \in O$
- O is closed under arbitrary unions and finite intersections

O is called the collection of *open sets* of the topological space

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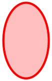
Example: Euclidian space (2D)

open set



closed set

- open balls (in $\mathbb{R}^n$) are open sets

- $\mathcal{I}^T(S)$ is the *largest open set* contained in S
- $\mathcal{C}^T(S)$ is the *smallest closed set* containing S

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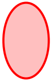

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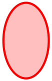

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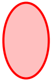

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Modal Logic

$$\Phi ::= p \mid \top \mid \bot \mid \neg\Phi \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \Box\Phi \mid \Diamond\Phi$$

A topological space (X, O)

- X a set of points
- O the set of open sets of X

A *model* $\mathcal{M} = ((X, O), \mathcal{V})$

- (X, O) a topological space
- $\mathcal{V} : P \to \mathcal{P}(X)$ a valuation function

$\mathcal{V}$ assigns to each atomic proposition the set of points that satisfy it.

$$\begin{array}{lcl}
\mathcal{M}, x \models \top & \iff & \textit{true} \\
\mathcal{M}, x \models p & \iff & x \in \mathcal{V}(p) \\
\mathcal{M}, x \models \neg\phi & \iff & \texttt{not}\, \mathcal{M}, x \models \phi \\
\mathcal{M}, x \models \phi \wedge \psi & \iff & \mathcal{M}, x \models \phi \texttt{ and } \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \Box\phi & \iff & \exists o \in O.(x \in o \texttt{ and } \forall y \in o. \mathcal{M}, y \models \phi) \\
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Modal Logic of Space [McKinsey & Tarski]

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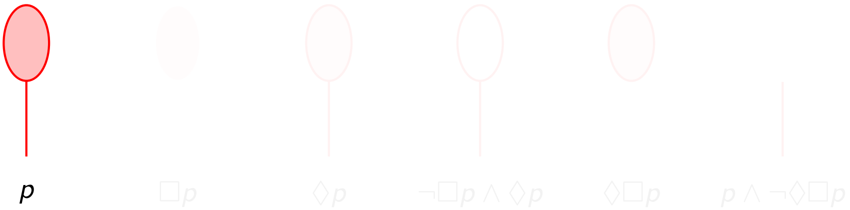

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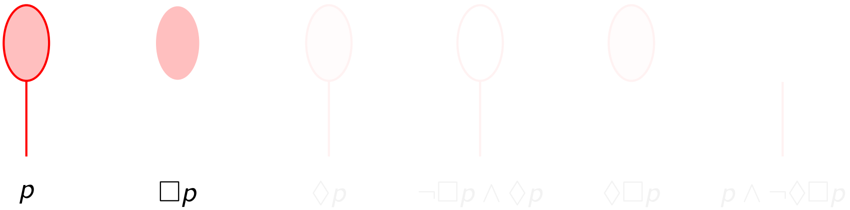

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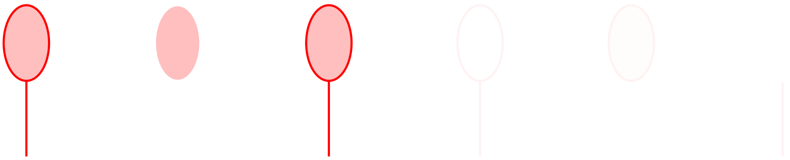

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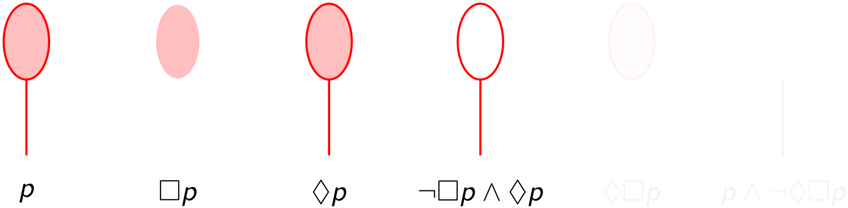

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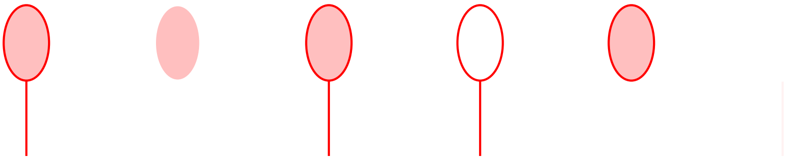

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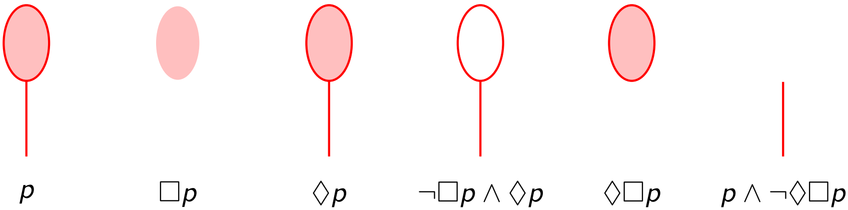

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Alternative characterisation of Topological Space
[Kuratowski]

A topological space is a pair $(X, \mathcal{C}^T)$ with $\mathcal{C}^T : 2^X \rightarrow 2^X$ such that

for each $A, B \subseteq X$:

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Interior and closure are duals:

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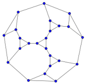
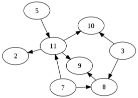

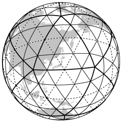
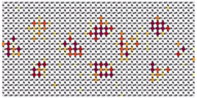
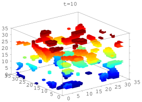

Unfortunately topological spaces work only
for discrete spaces that are *not* that much interesting
(e.g. empty or complete graphs)

We want to be able to deal also with

GENERIC GRAPHS

as models of space(s)

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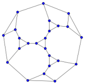

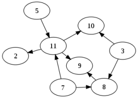

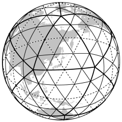

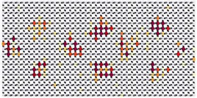

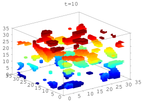

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Čech Spaces or Closure Spaces

A *closure space* is a pair $(X, \mathcal{C})$ with $\mathcal{C} : 2^X \to 2^X$ such that

for each $A, B \subseteq X$:

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- ~~$\mathcal{C}(\mathcal{C}(A)) = \mathcal{C}(A)$~~

Define:

- $\mathcal{I}(A) = \overline{\mathcal{C}(\overline{A})}$
- A is *open* iff $A = \mathcal{I}(A)$
- A is *closed* iff $A = \mathcal{C}(A)$
- A is a *neighbourhood* of $x \in X$ iff $x \in \mathcal{I}(A)$

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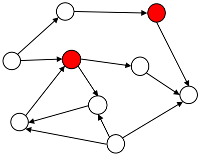
Graphs as Closure Spaces

A graph is a set of nodes X and a binary relation $R \subseteq X \times X$

$$\mathcal{C}_R(A) = A \cup \{x \in X | \exists a \in A.(a, x) \in R\}$$

The pair $(X, \mathcal{C}_R)$ is a closure space



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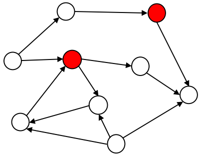

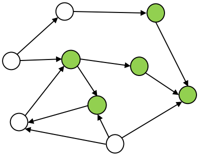

Quasi-discrete Closure Spaces

A closure space $(X, \mathcal{C})$ is *quasi-discrete* if and only if either one of the following holds:

- each $x \in X$ has a *minimal neighbourhood* N_x
- for each $A \subseteq X$, $\mathcal{C}(A) = \bigcup_{a \in A} \mathcal{C}(\{a\})$

A is a neighbourhood of $x \in X$ iff $x \in \mathcal{I}(A)$

Theorem

$(X, \mathcal{C})$ is quasi-discrete iff there is $R \subseteq X \times X$ such that $\mathcal{C} = \mathcal{C}_R$

Lemma

$\mathcal{C}_R$ is idempotent iff the reflexive closure $R^=$ of R is transitive

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A is a neighbourhood of $x \in X$ iff $x \in \mathcal{I}(A)$

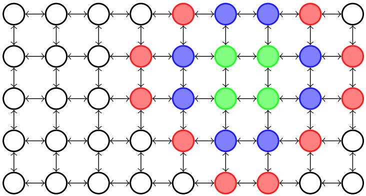
Theorem

$(X, \mathcal{C})$ is quasi-discrete iff there is $R \subseteq X \times X$ such that $\mathcal{C} = \mathcal{C}_R$

Lemma

$\mathcal{C}_R$ is idempotent iff the reflexive closure $R^=$ of R is transitive

Graphs inducing Quasi Discrete Closure Spaces



- $A = \{\bullet, \bullet\}$
- $\mathcal{I}(A) = \{\bullet\}$ and $\mathcal{C}(A) = \{\bullet, \bullet, \bullet\}$
- $\mathcal{B}(A) = \mathcal{C}(A) \setminus \mathcal{I}(A) = \{\bullet, \bullet\}$
- $\mathcal{B}^{-}(A) = A \setminus \mathcal{I}(A) = \{\bullet\}$
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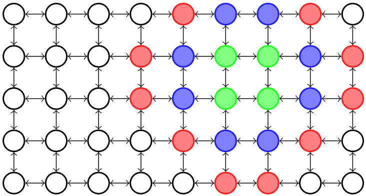
But also graphs with an uncountable set of nodes/points such as $(\mathbb{R}, \leq)$ are quasi-discrete closure spaces

Graphs inducing Quasi Discrete Closure Spaces



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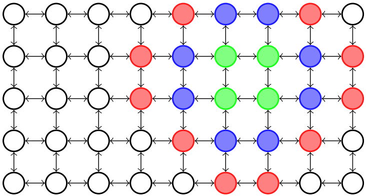

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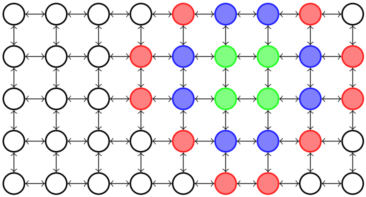

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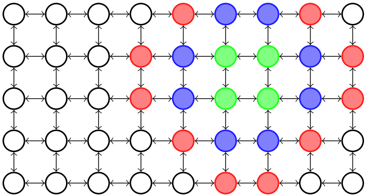

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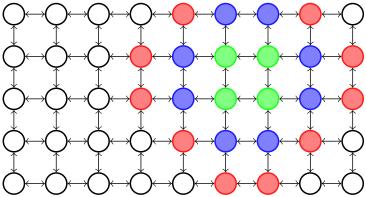

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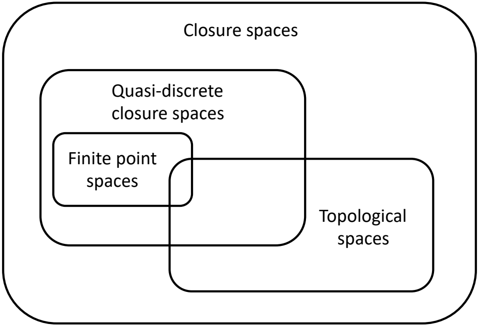
Hierarchy of Closure Spaces



PART II

Spatial Logic for Closure Spaces

Spatial Logic for Closure Spaces (SLCS)

..... a little alchemy ...

What if we interpret Temporal Logics operators (e.g. $\mathcal{U}$) on structures which represent space?

$$\Phi_1 \, \mathcal{U} \, \Phi_2$$

$$\Downarrow$$

The points in space which satisfy Φ_1 and are *surrounded* by points satisfying Φ_2

What if we interpret Temporal Logics operators (e.g. $\mathcal{U}$) on structures which represent space?

$$\Phi_1 \, \mathcal{U} \, \Phi_2$$

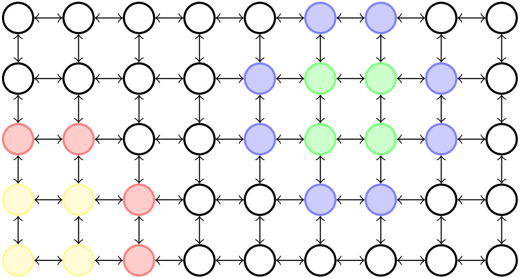
$$\Downarrow$$

The points in space which
satisfy Φ_1 and
are *surrounded* by points satisfying Φ_2

SLCS syntax

$$\begin{array}{llll} \Phi & ::= & p & [\textsc{Atomic proposition}] \\ & \mid & \top & [\textsc{True}] \\ & \mid & \neg\Phi & [\textsc{Not}] \\ & \mid & \Phi \wedge \Phi & [\textsc{And}] \\ & \mid & \mathcal{N}\Phi & [\textsc{Near}] \\ & \mid & \Phi\,\mathcal{S}\,\Phi & [\textsc{Surrounded}] \end{array}$$

Spatial operators: intuition

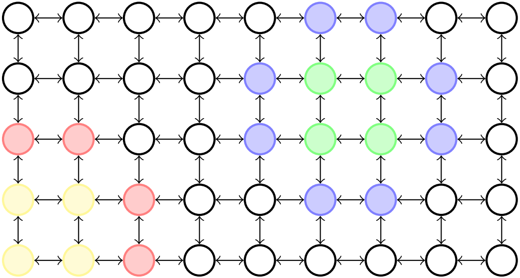


All red and yellow points satisfy $\mathcal{N}$ *yellow*

Green points satisfy *green* $\mathcal{S}$ *blue*

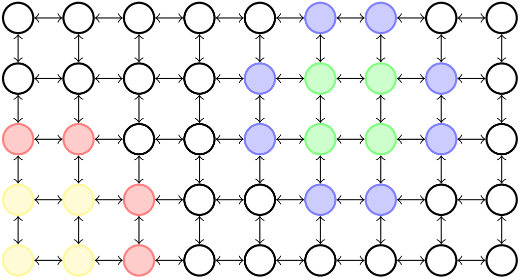
Yellow points satisfy *yellow* $\mathcal{S}$ *red*

Spatial operators: intuition



All red and yellow points satisfy $\mathcal{N}\, yellow$
Green points satisfy $green\, \mathcal{S}\, blue$
Yellow points satisfy $yellow\, \mathcal{S}\, red$

Spatial operators: intuition



All red and yellow points satisfy $\mathcal{N}$ *yellow*
Green points satisfy *green* $\mathcal{S}$ *blue*
Yellow points satisfy *yellow* $\mathcal{S}$ *red*

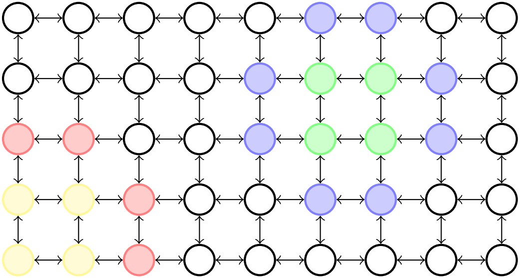
Semantics of SLCS

Satisfaction $\mathcal{M}, x \models \phi$ of formula ϕ at point x in quasi-discrete closure model $\mathcal{M} = ((X, \mathcal{C}), \mathcal{V})$ is defined, by induction on terms, as follows:

$$
\begin{array}{lcllll}
\mathcal{M}, x & \models & p & \iff & x \in \mathcal{V}(p) \\
\mathcal{M}, x & \models & \top & \iff & \mathit{true} \\
\mathcal{M}, x & \models & \neg\phi & \iff & \texttt{not}\, \mathcal{M}, x \models \phi \\
\mathcal{M}, x & \models & \phi \wedge \psi & \iff & \mathcal{M}, x \models \phi \;\texttt{and}\; \mathcal{M}, x \models \psi \\
\mathcal{M}, x & \models & \mathcal{N}\phi & \iff & x \in \mathcal{C}(\{y \in X | \mathcal{M}, y \models \phi\}) \\
\mathcal{M}, x & \models & \phi\, \mathcal{S}\, \psi & \iff & \exists A \subseteq X . x \in A \wedge \forall y \in A . \mathcal{M}, y \models \phi \wedge \\
 & & & & \quad \forall z \in \mathcal{B}^{+}(A) . \mathcal{M}, z \models \psi
\end{array}
$$

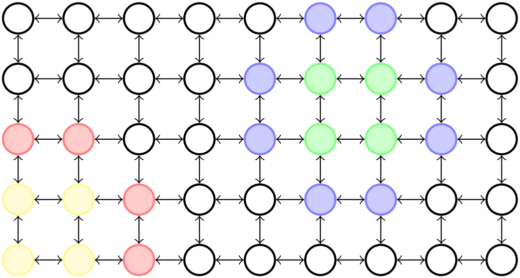
more . . .

Derived operators



$$
\begin{array}{llll}
\mathcal{I}\phi & \triangleq & \neg(\mathcal{N}\neg\phi) & [\textsc{interior}] \\
\delta\phi & \triangleq & (\mathcal{N}\phi) \wedge (\neg\mathcal{I}\phi) & [\textsc{boundary}] \\
\delta^{-}\phi & \triangleq & \phi \wedge (\neg\mathcal{I}\phi) & [\textsc{internal/interior boundary}] \\
\delta^{+}\phi & \triangleq & (\mathcal{N}\phi) \wedge (\neg\phi) & [\textsc{external/closure boundary}]
\end{array}
$$

Derived operators[1]



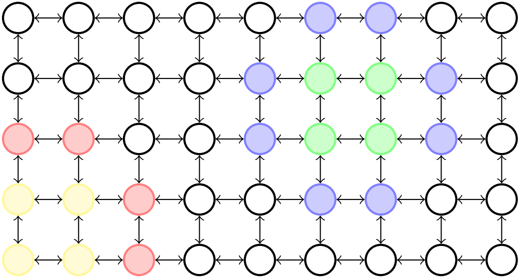
$$\begin{array}{llll} \mathcal{E}\phi & \triangleq & \phi\,\mathcal{S}\,\bot & [\textsc{everywhere}] \\ \mathcal{F}\phi & \triangleq & \neg\mathcal{E}(\neg\phi) & [\textsc{somewhere}] \end{array}$$

1 *[John H. Reif, A. Prasad Sistla, ICALP 1983]*

Derived operators

$$\phi\,\mathcal{R}\psi \quad \triangleq \quad \neg((\neg\psi)\,\mathcal{S}(\neg\phi)) \qquad [\text{REACHABILITY}]$$

$$\phi\,\mathcal{T}\,\psi \quad \triangleq \quad \phi \wedge ((\phi \vee \psi)\,\mathcal{R}\psi) \quad [\text{FROM-TO}]$$



$\phi\,\mathcal{R}\psi$: either ψ holds in x, or there exists a sequence of points after x, all satisfying ϕ leading to a point satisfying both ϕ and ψ

$(\textit{white} \vee \textit{blue})\,\mathcal{R}\,\textit{blue}$ satisfied by $\{\bullet, \bullet, \circ, \bullet\}$

$\textit{white}\,\mathcal{T}\,\textit{blue}$ satisfied by $\{\circ\}$

PART III

Model Checking Spatial Logics

Spatial Model checking (finite models)

Model checking in quasi-discrete closure spaces is analysis of a graph

Efficient algorithm $O(\mathit{nodes} + \mathit{arcs})$ for checking $\phi\,\mathcal{S}\,\psi$

Implemented as a “flooding” algorithm

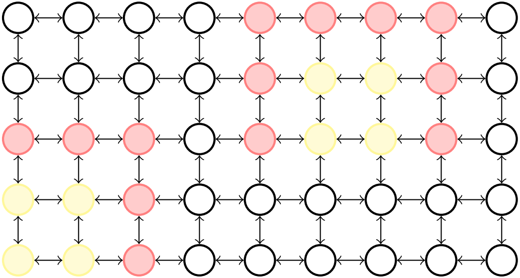
Efficient algorithm

The algorithm identifies *"bad"* areas, where $\neg\phi$ can be reached *without* passing by points satisfying ψ

Implemented recursively as an operator that enlarges the set of "bad" points at each application

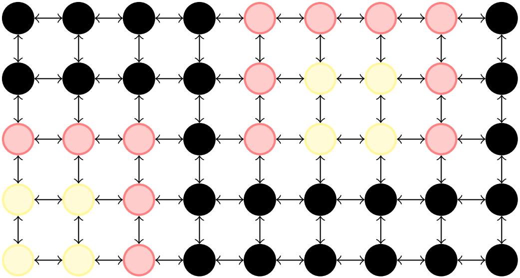
Upon fixed point: the points where ϕ holds, that are not "bad", satisfy $\phi \mathcal{S} \psi$.

Sat($\mathcal{M}$, *yellow* $\mathcal{S}$ *red*)



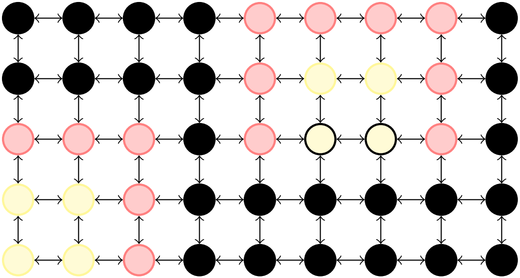
Find points satisfying *yellow* $\mathcal{S}$ *red*

Sat($\mathcal{M}$, *yellow* $\mathcal{S}$ *red*)



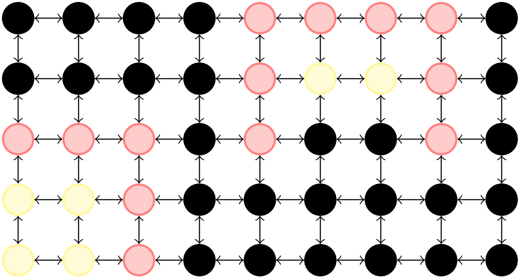
1) Find points satisfying neither *yellow* nor *red* and make them black

Sat($\mathcal{M}$, *yellow* $\mathcal{S}$ *red*)



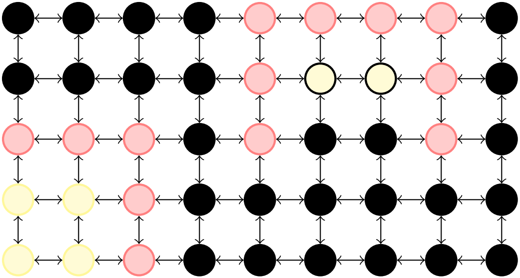
2) Identify yellow points in $\mathcal{C}$(*black*) . . .

Sat($\mathcal{M}$, *yellow* $\mathcal{S}$ *red*)



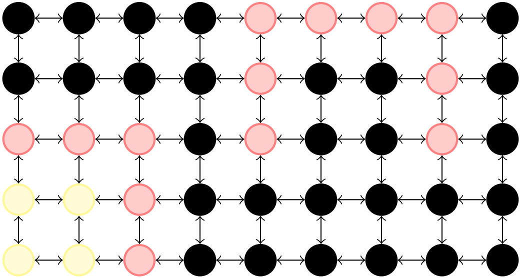
3) . . . and make them black

Sat($\mathcal{M}$, *yellow* $\mathcal{S}$ *red*)



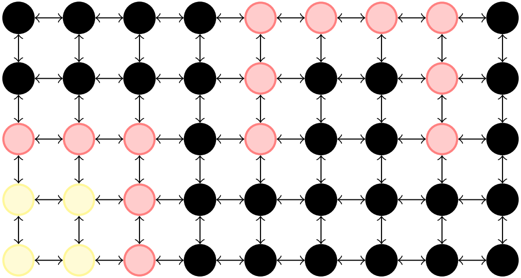
4) Identify yellow points in $\mathcal{C}$(*black*) . . .

Sat($\mathcal{M}$, *yellow* $\mathcal{S}$ *red*)



5) . . . and make them black

Sat($\mathcal{M}$, *yellow* $\mathcal{S}$ *red*)



Fixed point reached, the yellow points satisfy *yellow* $\mathcal{S}$ *red*

Model Checking Algorithm

Function `Sat`$(\mathcal{M}, \phi)$
    Input: Finite, quasi-discrete closure model
        $\mathcal{M} = ((X, \mathcal{C}), \mathcal{V})$, formula ϕ
    Output: Set of points $\{x \in X \mid \mathcal{M}, x \models \phi\}$
    Match ϕ
        case $\top$: **return** X
        case p : **return** $\mathcal{V}(p)$
        case $\neg\phi_1$:
            let $P =$ `Sat`$(\mathcal{M}, \phi_1)$
            return $X \setminus P$
        case $\phi_1 \wedge \phi_2$:
            let $P =$ `Sat`$(\mathcal{M}, \phi_1)$
            let $Q =$ `Sat`$(\mathcal{M}, \phi_2)$
            return $P \cap Q$
        case $\mathcal{N}\phi_1$:
            let $P =$ `Sat`$(\mathcal{M}, \phi_1)$
            return $\mathcal{C}(P)$
        case $\phi_1 \mathcal{S} \phi_2$:
            return `CheckSurr` $(\mathcal{M},\phi_1,\phi_2)$

Function `CheckSurr` $(\mathcal{M},\phi_1,\phi_2)$
    Input: Finite, quasi-discrete closure model
        $\mathcal{M} = ((X, \mathcal{C}), \mathcal{V})$, formulas ϕ_1, ϕ_2
    Output: Set of points $\{x \in X \mid \mathcal{M}, x \models \phi_1 \mathcal{S} \phi_2\}$
    var $V :=$ `Sat`$(\mathcal{M}, \phi_1)$
    let $Q =$ `Sat`$(\mathcal{M}, \phi_2)$
    var $T := \mathcal{B}^+(V \cup Q)$
    while $T \neq \emptyset$ **do**
        var $T' := \emptyset$
        for $x \in T$ **do**
            let $N = pre(x) \cap V$
            $V := V \setminus N$
            $T' := T' \cup (N \setminus Q)$
        $T := T'$;
    return V

Correctness and Complexity

Theorem

For any finite quasi-discrete closure model $\mathcal{M} = ((X, \mathcal{C}), \mathcal{V})$ and SLCS formula ϕ, $x \in \mathtt{Sat}(\mathcal{M}, \phi)$ if and only if $\mathcal{M}, x \models \phi$

Proposition

For any finite quasi-discrete model $\mathcal{M} = ((X, \mathcal{C}_R), \mathcal{V})$ and SLCS formula ϕ of size k, function $\mathtt{Sat}(\mathcal{M}, \phi)$ terminates in $\mathcal{O}(k \cdot (|X| + |R|))$ steps

PART IV

Some applications

Some Recent Results[2]

Theory, Algorithms and Tools:

- closure spaces: graphs, images (based on Galton's work)
- new operators: *reach, surrounded, touch, ...*
- topochecker: spatio-temporal & collective model checking
- topochecker + MultiVeSTa: statistical spatio-temporal MC
- `topochecker.isti.cnr.it`
- VoxLogica: image analysis
- `github.com/vincenzoml/VoxLogicA`

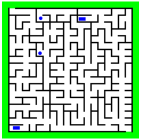
Applications:

- smart transportation (bike sharing, buses, train control);
- image analysis (medical domain)

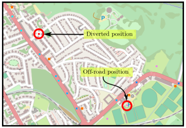
---

[2] *[Ciancia, Latella, Loreti, Massink - LMCS 2016]*
[Ciancia, Latella, Massink, Paškauskas, Vandin, ISoLA 2016]
[Ciancia, Gilmore, Grilletti, Latella, Loreti, Massink, STTT 2018]
[Banci Buonamici, Belmonte, Ciancia, Latella, Massink, STTT 2019]
[Ciancia, Belmonte, Latella, Massink, TACAS 2019]

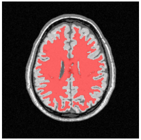
Selected Applications



A maze



Smart buses GPS



Medical Imaging

London bike sharing

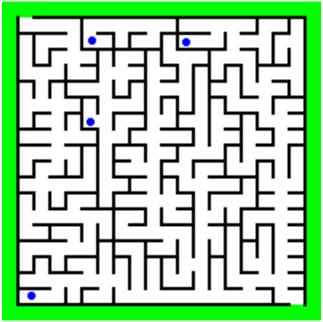
Turing patterns

Embedding RCC8D

Digital images

Any digital image can be treated as a finite, quasi discrete, closure space

Atomic propositions: white, green, black, blue



toExit = [white] T [green] {●}

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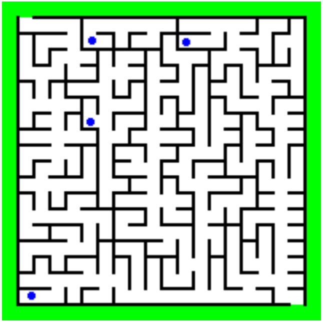

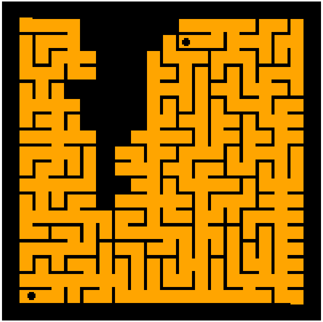

toExit = [white] T [green] {●}

fromStartToExit = toExit & ([white] T [blue]) {●}

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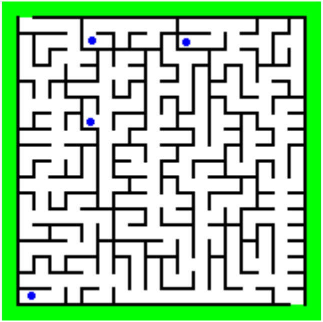 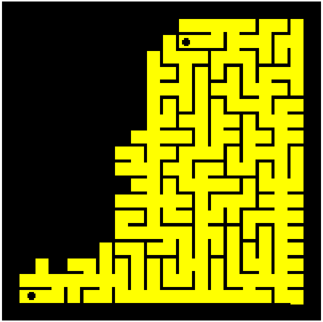

toExit = [white] T [green] {●}

fromStartToExit = toExit & ([white] T [blue]) {●}

startCanExit = [blue] T fromStartToExit {●}

back

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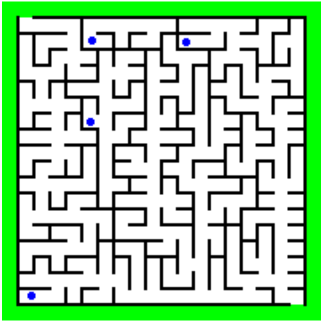
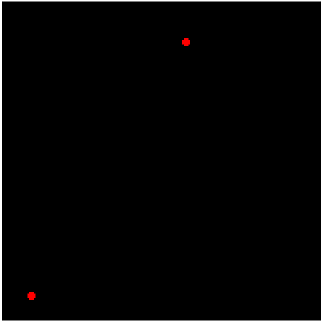




```
toExit = [white] T [green] {●}

fromStartToExit = toExit & ([white] T [blue]) {●}

startCanExit = [blue] T fromStartToExit {●}
```

back

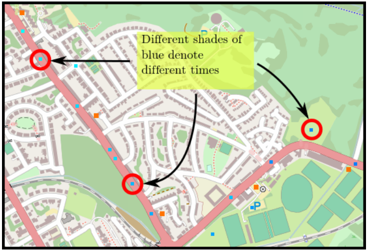
Implausible data in GPS traces of Edinburgh buses

Spatial ordering of data points

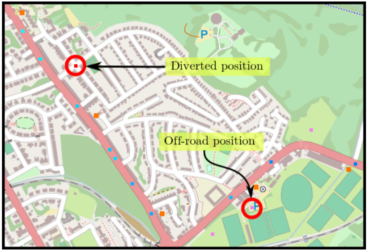
"not on a main street"

"not on a street at all"

[Ciancia, Gilmore, Grilletti et al., STTT 2018]



spatial model



model checking result

back

Medical Image Analysis: **ImgQL**

[Banci Buonamici, Belmonte, Ciancia, Latella, Massink, STTT 2019]
[Ciancia, Belmonte, Latella, Massink, TACAS 2019]

ImgQL variant of SLCS

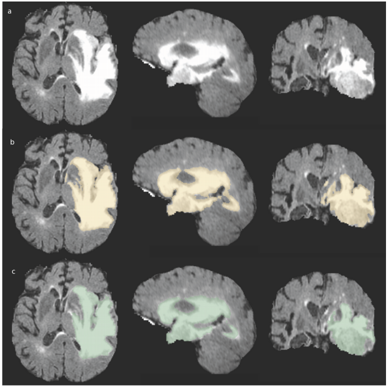
$$\Phi ::= p \mid \neg\,\Phi \mid \Phi_1 \vee \Phi_2 \mid \mathcal{N}\Phi \mid \Phi_1\ \mathcal{S}\ \Phi_2 \mid \mathcal{D}^I\ \Phi$$

Derived:

- Surrounded
- Region Growing

Domain specific:

- Distance Operator
- Statistical Texture Similarity Operator
- Percentiles
- Tool: VoxLogicA



GTV for TCIA 471 patient from BraTS 2017 dataset

`https://github.com/vincenzoml/VoxLogicA`

Medical Image Analysis: **ImgQL**

[Banci Buonamici, Belmonte, Ciancia, Latella, Massink, STTT 2019]
[Ciancia, Belmonte, Latella, Massink, TACAS 2019]

ImgQL variant of SLCS

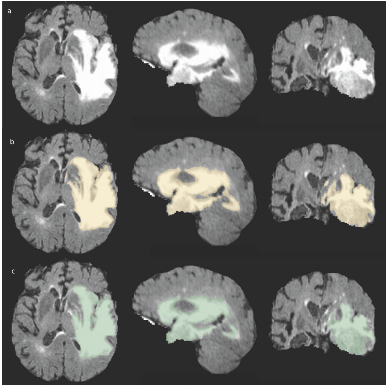
$$\Phi ::= p \mid \neg\Phi \mid \Phi_1 \vee \Phi_2 \mid \mathcal{N}\Phi \mid \rho\,\Phi_2[\Phi_1] \mid \mathcal{D}^I\,\Phi$$

Derived:

- Surrounded
- Region Growing

Domain specific:

- Distance Operator
- Statistical Texture Similarity Operator
- Percentiles
- Tool: VoxLogicA



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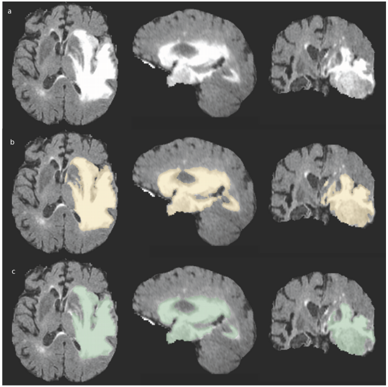

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Derived:

- $\Phi_1 \; \mathcal{S} \; \Phi_2 \triangleq \Phi_1 \wedge \neg\rho\,(\neg(\Phi_1 \vee \Phi_2))[\neg\Phi_2]$
- $\mathit{grow}(\Phi_1, \Phi_2) \triangleq \Phi_1 \vee \mathit{touch}(\Phi_2, \Phi_1)$

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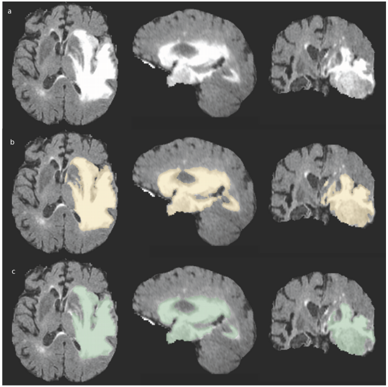

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Domain Specific Operators

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Statistical Texture Similarity Operator

A point x satisfies $\triangle_{\bowtie c}\begin{bmatrix} m & M & k \\ r & a & b \end{bmatrix}\Phi$ iff, letting h_a be the histogram of the *sphere* of radius r centred in x and h_b that of the Φ-area, we have $cross\text{–}correlation(h_a, h_b) \bowtie c$

White matter[3]**:**

original MRI

[3] Original MRI: Pat04 from *[Aubert-Broche et al. IEEE Trans. on Med. Im., 25(11), 2006]*

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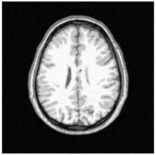

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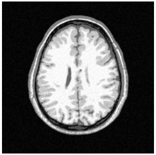

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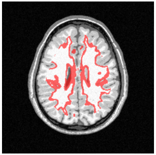

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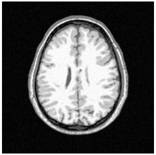

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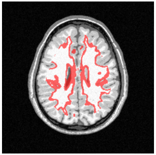

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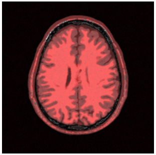

similarity score

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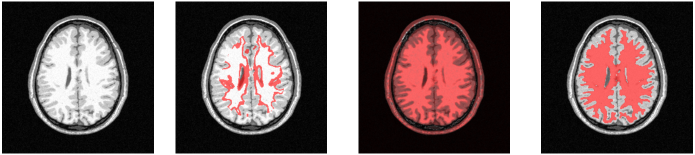
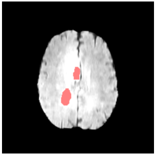


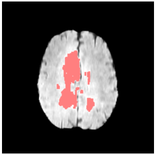
original MRI | likely white | similarity score | highly similar

[3] Original MRI: Pat04 from *[Aubert-Broche et al. IEEE Trans. on Med. Im., 25(11), 2006]*

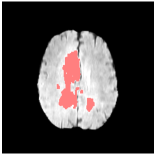
3D Magnetic Resonance Tumour Segmentation[4,5]



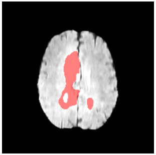
hyper intense (hI)



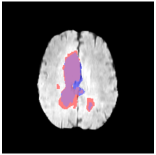
very intense (vI)



grow(hI,vI) (c)



similar texture (d)



gtv=grow(c,d) manual (blue)

4 *[Belmonte,Ciancia,Latella,Massink,TACAS19]*

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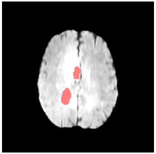

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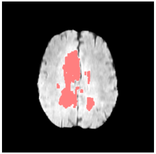

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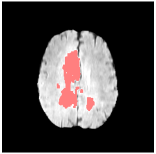

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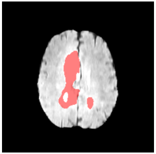

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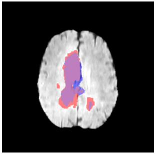

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Brain Tumor Image Segmentation Benchmark (BraTS) 2017

Comparison of 18 BraTS17 techniques that analyse at least 100 cases:
Similarity score (Dice GTV): 0.88 (avg.) 0.64-0.96 (range)

more . . .

Our score on 193 cases: 0.85 (avg.) 0.10 (std.)

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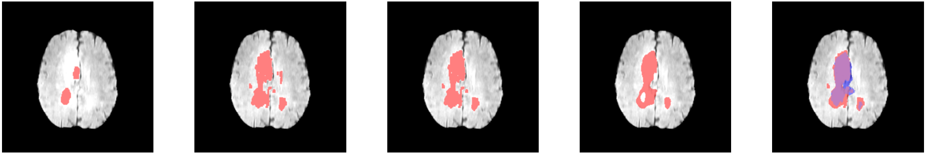

hyper intense (hI) | very intense (vI) | grow(hI,vI) (c) | similar texture (d) | gtv=grow(c,d) manual (blue)

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About 10 seconds on Intel Core I7 7700 (8 cores), $\sim$ 9 million voxels

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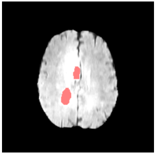

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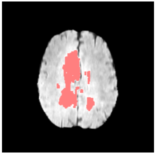

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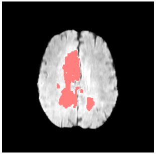

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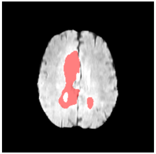

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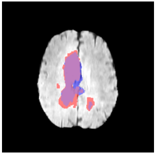

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more . . .

Our score on 193 cases: 0.85 (avg.) 0.10 (std.) In line with state-of-the-art!

About 10 seconds on Intel Core I7 7700 (8 cores), $\sim$ 9 million voxels

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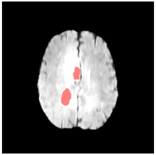

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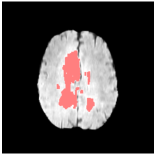

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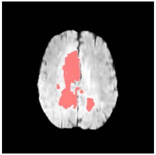

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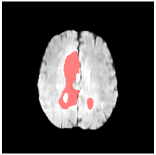

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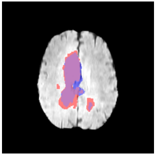

gtv=grow(c,d) manual (blue)

```
let background = touch(intensity <.  0.1,border)
let brain = !background

let pflair = percentiles(intensity,brain)
let hI = pflair >.  0.95
let vI = pflair >.  0.86
let hyperIntense = flt(5.0,hI)
let veryIntense = flt(2.0,vI)

let growTum = grow(hyperIntense,veryIntense)
let tumSim = similarTo(growTum)
let tumStatCC = flt(2.0,tumSim >.  0.6)
let gtv = grow(growTum,tumStatCC)
```

background removal

threshholding

region growing and texture similarity

back

4 *[Belmonte,Ciancia,Latella,Massink,TACAS19]*

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Spatio-Temporal Logics (SLCS+CTL)

Syntax

$$
\begin{array}{rcll}
\Phi & ::= & \top & [\textsc{True}] \\
& | & p & [\textsc{Atomic proposition}] \\
& | & \neg\Phi & [\textsc{Not}] \\
& | & \Phi \vee \Phi & [\textsc{Or}] \\
& | & \mathcal{N}\Phi & [\textsc{Near}] \\
& | & \Phi\,\mathcal{S}\,\Phi & [\textsc{Surrounded}]
\end{array}
\Bigg\}\ \textit{Spatial}
$$

$$
\begin{array}{rcll}
& | & \mathtt{A}\varphi & [\textsc{All Paths}] \\
& | & \mathtt{E}\varphi & [\textsc{Exist Path}]
\end{array}
\Big\}\ \textit{Temporal}
$$

$$
\begin{array}{rcll}
\varphi & ::= & \mathcal{X}\Phi & [\textsc{Next}] \\
& | & \Phi\,\mathcal{U}\,\Psi & [\textsc{Until}]
\end{array}
\Big\}\ \textit{Path formulas}
$$

Spatio-Temporal Logics (STLCS)

Semantics

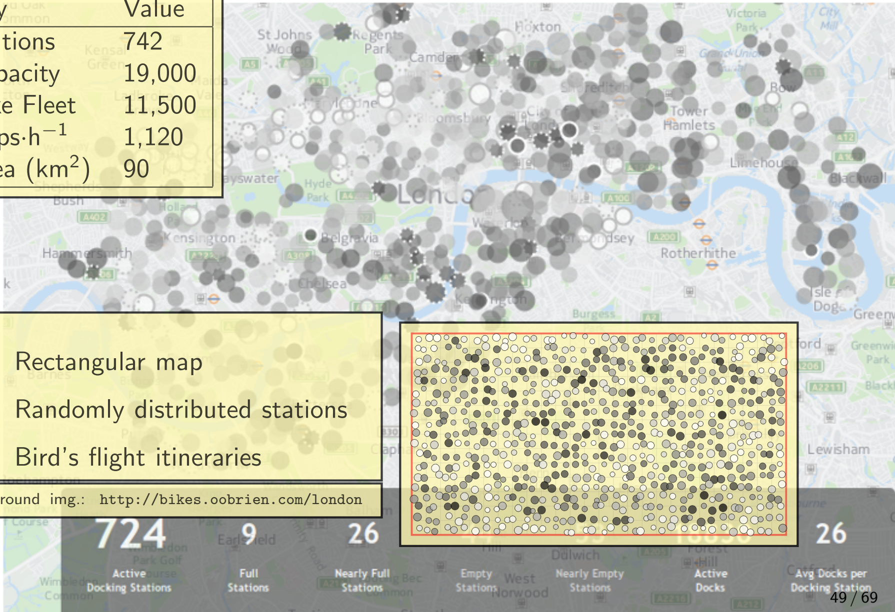
Satisfaction $\mathcal{M}, x, s \models \Phi$ of an STLCS formula Φ at point x and state s in model $\mathcal{M} = ((X, \mathcal{C}), (S, R), \mathcal{V}_{s \in S})$ is defined as follows:

$$
\begin{array}{lcllll}
\mathcal{M}, x, s & \models & \top & & \\
\mathcal{M}, x, s & \models & p & \Leftrightarrow & x \in \mathcal{V}_s(p) \\
\mathcal{M}, x, s & \models & \neg\Phi & \Leftrightarrow & \mathcal{M}, x, s \not\models \Phi \\
\mathcal{M}, x, s & \models & \Phi \vee \Psi & \Leftrightarrow & \mathcal{M}, x, s \models \Phi \text{ or } \mathcal{M}, x, s \models \Psi \\
\mathcal{M}, x, s & \models & \mathcal{N}\Phi & \Leftrightarrow & x \in \mathcal{C}(\{y \in X | \mathcal{M}, y, s \models \Phi\}) \\
\mathcal{M}, x, s & \models & \Phi \, \mathcal{S} \, \Psi & \Leftrightarrow & \exists A \subseteq X . x \in A \wedge \forall y \in A . \mathcal{M}, y, s \models \Phi \wedge \\
 & & & & \quad \wedge \forall z \in \mathcal{B}^+(A) . \mathcal{M}, z, s \models \Psi \\
\mathcal{M}, x, s & \models & \mathtt{A}\,\varphi & \Leftrightarrow & \forall \sigma \in \mathcal{P}_s . \, \mathcal{M}, x, \sigma \models \varphi \\
\mathcal{M}, x, s & \models & \mathtt{E}\,\varphi & \Leftrightarrow & \exists \sigma \in \mathcal{P}_s . \, \mathcal{M}, x, \sigma \models \varphi \\
\mathcal{M}, x, \sigma & \models & \mathcal{X}\Phi & \Leftrightarrow & \mathcal{M}, x, \sigma(1) \models \Phi \\
\mathcal{M}, x, \sigma & \models & \Phi \, \mathcal{U} \, \Psi & \Leftrightarrow & \exists n . \, \mathcal{M}, x, \sigma(n) \models \Psi \text{ and} \\
 & & & & \forall n' \in [0, n) . \mathcal{M}, x, \sigma(n') \models \Phi
\end{array}
$$

Bike sharing: Clusters of full docking stations

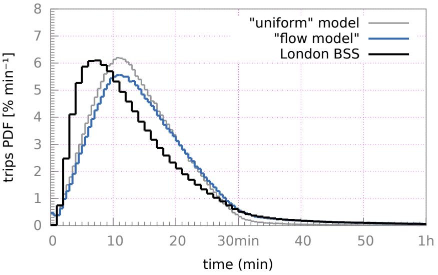
[Ciancia et al, SEFMWS15],[Massink, Paškauskas, ITSC15]

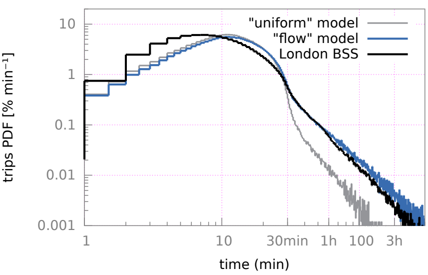
Key	Value
Stations	742
Capacity	19,000
Bike Fleet	11,500
Trips$\cdot$h^{-1}	1,120
Area (km^2)	90



- Rectangular map
- Randomly distributed stations
- Bird's flight itineraries

Background img.: `http://bikes.oobrien.com/london`

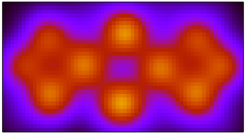




Expected Trips(>30)min$=0\%$

Uniform Multi-agent, uniform OD

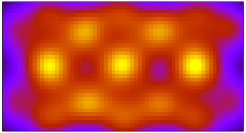
- Trips(>30)min$=2\%$



hiring probabilities

Flow Multi-agent, non-uniform OD

- Trips(>30)min$=7.7\%$ Bingo!



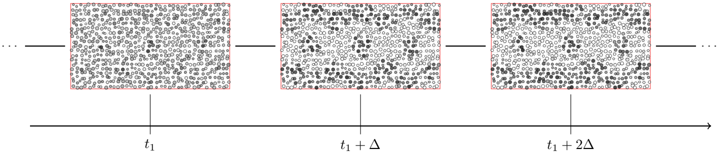
Soft control: dissolve clusters

returning probabilities

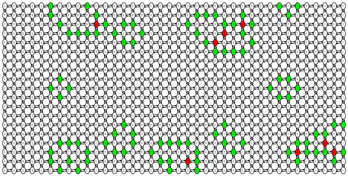
STLCS: Spatio-temporal MC

[Ciancia et al, SEFMWS15],[Massink, Paškauskas, ITSC15]

Detecting the emergence of clusters of full stations



- Define cluster: `cluster = I(full)`
- Cluster boundary: `(!EF cluster) & (N EF cluster)`



[topochecker, www.github.com/vincenzoml/topochecker]

Spatio-Temporal Logics (SLCS+CTL)

Syntax

Φ	$::=$	$\top$	[True]	*Spatial*
	$\mid$	p	[Atomic proposition]	
	$\mid$	$\neg\Phi$	[Not]	
	$\mid$	$\Phi \vee \Phi$	[Or]	
	$\mid$	$\mathcal{N}\Phi$	[Near]	
	$\mid$	$\Phi\,\mathcal{S}\,\Phi$	[Surrounded]	
	$\mid$	$\mathtt{A}\varphi$	[All Paths]	*Temporal*
	$\mid$	$\mathtt{E}\varphi$	[Exist Path]	
φ	$::=$	$\mathcal{X}\Phi$	[Next]	*Path formulas*
	$\mid$	$\Phi\,\mathcal{U}\,\Psi$	[Until]	

Spatio-Temporal Logics (STLCS)

Semantics

Satisfaction $\mathcal{M}, x, s \models \Phi$ of an STLCS formula Φ at point x and state s in model $\mathcal{M} = ((X, \mathcal{C}), (S, R), \mathcal{V}_{s \in S})$ is defined as follows:

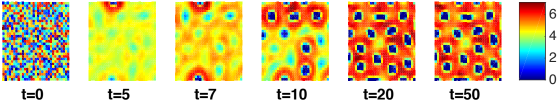
$$
\begin{array}{lcllll}
\mathcal{M}, x, s & \models & \top & & \\
\mathcal{M}, x, s & \models & p & \Leftrightarrow & x \in \mathcal{V}_s(p) \\
\mathcal{M}, x, s & \models & \neg \Phi & \Leftrightarrow & \mathcal{M}, x, s \not\models \Phi \\
\mathcal{M}, x, s & \models & \Phi \vee \Psi & \Leftrightarrow & \mathcal{M}, x, s \models \Phi \text{ or } \mathcal{M}, x, s \models \Psi \\
\mathcal{M}, x, s & \models & \mathcal{N} \Phi & \Leftrightarrow & x \in \mathcal{C}(\{y \in X | \mathcal{M}, y, s \models \Phi\}) \\
\mathcal{M}, x, s & \models & \Phi \, \mathcal{S} \, \Psi & \Leftrightarrow & \exists A \subseteq X . x \in A \wedge \forall y \in A . \mathcal{M}, y, s \models \Phi \wedge \\
 & & & & \quad \wedge \forall z \in \mathcal{B}^+(A) . \mathcal{M}, z, s \models \Psi \\
\mathcal{M}, x, s & \models & \mathtt{A} \, \varphi & \Leftrightarrow & \forall \sigma \in \mathcal{P}_s . \, \mathcal{M}, x, \sigma \models \varphi \\
\mathcal{M}, x, s & \models & \mathtt{E} \, \varphi & \Leftrightarrow & \exists \sigma \in \mathcal{P}_s . \, \mathcal{M}, x, \sigma \models \varphi \\
\mathcal{M}, x, \sigma & \models & \mathcal{X} \Phi & \Leftrightarrow & \mathcal{M}, x, \sigma(1) \models \Phi \\
\mathcal{M}, x, \sigma & \models & \Phi \, \mathcal{U} \, \Psi & \Leftrightarrow & \exists n . \, \mathcal{M}, x, \sigma(n) \models \Psi \text{ and} \\
 & & & & \forall n' \in [0, n) . \mathcal{M}, x, \sigma(n') \models \Phi
\end{array}
$$

Spatio-temporal analysis of Turing patterns (SSTL)

[Nenzi, Bortolussi, Latella, Loreti, Massink, RV15 + LMCS 2018]

Morphogenesis: Two chemical substances A and B in a $K \times K$ grid

$$\begin{cases} \frac{dx^A_{i,j}}{dt} = R_1 x^A_{i,j} x^B_{i,j} - x^A_{i,j} + R_2 + D_1(\mu^A_{i,j} - x^A_{i,j}) \\ \frac{dx^B_{i,j}}{dt} = R_3 x^A_{i,j} x^B_{i,j} + R_4 + D_2(\mu^B_{i,j} - x^B_{i,j}) \end{cases}$$



$$\phi_{\text{pattern}} := \mathcal{F}_{[T_{\text{pattern}}, T_{\text{pattern}}+\delta]} \mathcal{G}_{[0, T_{\text{end}}]}((x^A \le h) \mathcal{S}_{[w_1, w_2]}(x^A > h))$$

Detecting emergent spots and their persistence in time, including their robustness to small perturbations

back

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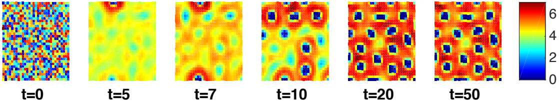

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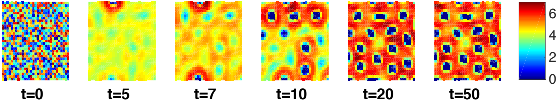

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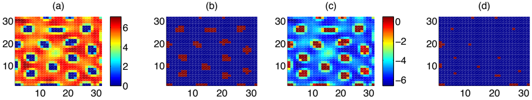
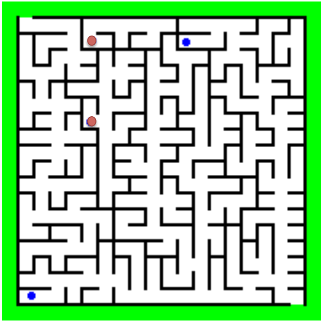


Detecting emergent spots and their persistence in time, including their robustness to small perturbations

back

Collective Spatial Logic[6]



model

The sets of points in blue can collectively reach an exit

6 *[Ciancia, Latella, Loreti, Massink, LMCS 12(4:2), 2016]*

Collective Spatial Logic

Syntax

Φ	$::=$	p	[Atomic proposition]
	$\mid$	$\top$	[True]
	$\mid$	$\neg\Phi$	[Not]
	$\mid$	$\Phi \wedge \Phi$	[And]
	$\mid$	$\mathcal{N}\Phi$	[Near]
	$\mid$	$\Phi\,\mathcal{S}\,\Phi$	[Surrounded]

Ψ	$::=$	$\neg\,\Psi$	[Collective Not]
	$\mid$	$\Psi_1 \wedge \Psi_2$	[Collective And]
	$\mid$	$\Phi \prec \Psi$	[Share]
	$\mid$	$\mathcal{G}\Phi$	[Group]

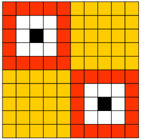
Collective Spatial Logic
Semantics

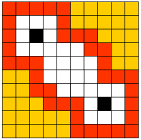
Satisfaction $\mathcal{M}, Y \models_C \Psi$ of a collective formula Ψ at set $Y \subseteq X$ in model $\mathcal{M} = ((X, \mathcal{C}), \mathcal{V})$ is defined by induction on the structure of formulas:

$$
\begin{array}{llll}
\mathcal{M}, Y & \models_C & \neg \Psi & \Leftrightarrow \mathcal{M}, Y \models_C \Psi \text{ does not hold} \\
\mathcal{M}, Y & \models_C & \Psi_1 \wedge \Psi_2 & \Leftrightarrow \mathcal{M}, Y \models_C \Psi_1 \text{ and } \mathcal{M}, Y \models_C \Psi_2 \\
\mathcal{M}, Y & \models_C & \Phi \prec \Psi & \Leftrightarrow \mathcal{M}, \{x \in Y | \mathcal{M}, x \models \Phi\} \models_C \Psi \\
\mathcal{M}, Y & \models_C & \mathcal{G}\,\Phi & \Leftrightarrow \text{there exists } Z \subseteq X \text{ such that} \\
 & & & \quad Y \subseteq Z \text{ and } Z \text{ is path-connected and} \\
 & & & \quad \text{for all } z \in Z \text{ we have: } \mathcal{M}, z \models \Phi
\end{array}
$$

Collective Spatial Logic

Simple example

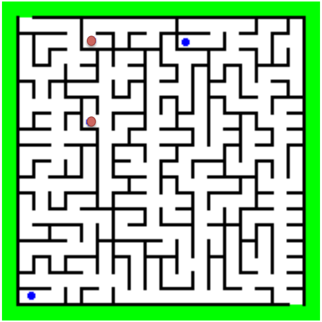




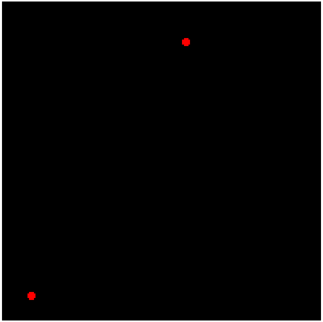
Φ: $(\mathit{black} \vee \mathit{white})\,\mathcal{S}\,\mathit{red}$

$\mathcal{M}$, $\{y | y \text{ is black}\} \models_C \mathcal{G}(\Phi)$

Collective Spatial Logic



model



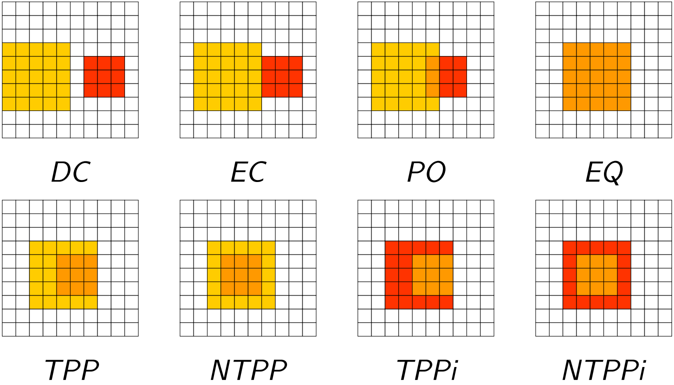
result

The set of blue points can collectively reach an exit

$\mathcal{M}, \{y | y \text{ is blue}\} \models_C \mathcal{G}(\texttt{white} \vee \texttt{startCanExit})\ \{\bullet\}$

Embedding of Discrete Region Connection Calculus (RCC8D)[7]

[Randell, Cui, Cohn, KR'92, 1992]



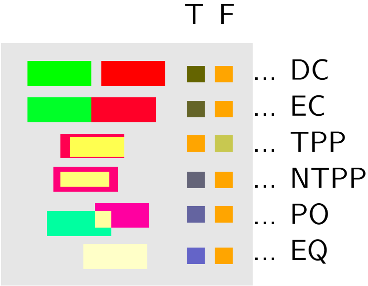
more . . .

[7] *[Ciancia, Latella, Massink, LNCS 11665, 2019]*

Embedding RCC8D in CSLCS[8]

Verification with topochecker

TPP(*Green*, *Red*)



Produced using the *spatio-temporal* model-checker `topochecker`

`http://topochecker.isti.cnr.it/`

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8 *[Ciancia, Latella, Massink, LNCS 11665, 2019]*

Conclusions and Outlook

"Nothing is more practical than a good theory"[9]

Future work:

- Spatial Model Reduction
- Spatial Monitoring and Spatial Computing
- Medical Imaging
- Data and Topology

[9] Kurt Lewin, 1951

Thanks for listening!

Hope you enjoyed your travel through space!

Selected References

Banci19 Banci Buonamici, F., Belmonte, G., Ciancia, V., Latella, D., Massink, M.: Spatial logics and model checking for medical imaging. In: STTT, doi: 10.1007/s10009-019-00511-9 (2019)

Belmonte19 Belmonte, G., Ciancia, V., Latella, D., Massink, M.:VoxLogicA: A Spatial Model Checker for Declarative Image Analysis. TACAS 2019. LNCS vol. 11427 (2019)

Benthem07 van Benthem, J., Bezhanishvili, G.: Modal logics of space. In: Handbook of Spatial Logics, pp. 217–298. Springer (2007)

Ciancia14 Ciancia, V., Latella, D., Loreti, M., Massink, M.: Specifying and Verifying Properties of Space. In: The 8th IFIP International Conference on Theoretical Computer Science, TCS 2014, Track B. LNCS vol. 8705 (2014)

Ciancia16 Ciancia, V., Latella, D., Loreti, M., Massink, M.: Model Checking Spatial Logics for Closure Spaces. In: Logical Methods in Computer Science 12(4) (2016)

Ciancia15/18 Ciancia, V., Grilletti, G., Latella, D., Loreti, M., Massink, M.: Spatio-temporal model checking of vehicular movement in public transport systems. In: STTT 20(3): 289-311 (2018). (See also: LNCS, vol. 9509, Springer (2015))

Galton03 Galton, A.: A generalized topological view of motion in discrete space. Theoretical Computer Science 305(1–3), 111 – 134 (2003),

Gol14 Aydin Gol, E., Bartocci, E., Belta, C.: A formal methods approach to pattern synthesis in reaction diffusion systems. In: Proc. of CDC (2014)

Nenzi15/18 Nenzi, L., Bortolussi, L., Ciancia, V., Loreti, M., Massink, M.: Qualitative and quantitative monitoring of spatio-temporal properties. In: Bartocci, E., Majumdar, R. (eds.) Runtime Verification RV15, LNCS, vol. 9333, Springer (2015) (See also: Logical Methods in Computer Science 14(4), 2018)

Smyth07 Smyth, M. B., Webster, J.: Discrete Spatial Models. In: Handbook of Spatial Logics, pp. 713–798. Springer (2007)

Turing1952 Turing, A.M.: The Chemical Basis of Morphogenesis. Philosophical Transactions of the Royal Society of London B: Biological Sciences (1952)

Vieu1997 Vieu, L. Spatial Representation and Reasoning in Artificial Intelligence. In: O. Stock (ed.) Spatial and Temporal Reasoning. Dordrecht: Kluwer, (1997)

Surrounded versus Until

$$\Phi_2 \vee (\Phi_1 \ \mathcal{S} \ \Phi_2) \equiv \mathtt{A}(\Phi_1 \, \mathcal{W} \, \Phi_2)$$

where:

- $\mathtt{A}$ is the *path universal quantifier*
- $\mathcal{W}$ the weak-until operator

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Similarity indexes in Medical Imaging

$$Dice = 2 * \mathrm{TP}/(2 * \mathrm{TP} + \mathrm{FN} + \mathrm{FP})$$

with

TP = True Positive
FN = False Negative
FP = False Positive

Sensitivity is the fraction of True Positives:

$$\mathrm{Sens} = TP/(TP + FP)$$

Specificity is the fraction of True Negatives:

$$\mathrm{Spec} = TN/(TN + FN)$$

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Embedding RCC8D in CSLCS

Let $(X, \mathcal{C})$ a closure space and $\mathcal{M} = ((X, \mathcal{C}), \mathcal{V})$ a finite model.
Predicate p_Y denotes the set $Y \subseteq X$ s.t. $\mathcal{V}(p_Y) = Y$.
Encoding of standard set-theoretic and closure operators in CSLCS:

$$
\begin{array}{lcll}
[\![Y]\!] & = & p_Y, \text{for all } Y \subseteq X & [\textsc{Constant}] \\
[\![\overline{\gamma}]\!] & = & \neg[\![\gamma]\!] & [\textsc{Complement}] \\
[\![\gamma_1 \cap \gamma_2]\!] & = & [\![\gamma_1]\!] \wedge [\![\gamma_2]\!] & [\textsc{Intersection}] \\
[\![\mathcal{C}(\gamma)]\!] & = & \mathcal{N}([\![\gamma]\!]) & [\textsc{Closure}]
\end{array}
$$

where $\gamma, \gamma_1, \gamma_2$ range over expressions on sets built out of constants, complement, intersection and closure

Embedding RCC8D in CSLCS

Tests on the empty set, on set-inclusion and set-equality:

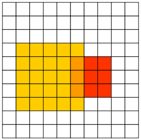
$$\begin{array}{lcll}
[\![\gamma = \emptyset]\!] & = & [\![\gamma]\!] \prec \mathcal{G}\bot & [\textsc{Empty}] \\
[\![\gamma_1 \subseteq \gamma_2]\!] & = & [\![(\gamma_1 \cap \overline{\gamma_2}) = \emptyset]\!] & [\textsc{Inclusion}] \\
[\![\gamma_1 = \gamma_2]\!] & = & [\![\gamma_1 \subseteq \gamma_2]\!] \wedge [\![\gamma_2 \subseteq \gamma_1]\!] & [\textsc{Equality}]
\end{array}$$

Embedding RCC8D in CSLCS

$$\begin{array}{lcll} [\![\mathtt{P}(Y_1, Y_2)]\!] & = & [\![Y_1 \subseteq Y_2]\!] \wedge \neg [\![Y_1 = \emptyset]\!] & [\textsc{Parthood}] \\ [\![\mathtt{O}(Y_1, Y_2)]\!] & = & \neg [\![Y_1 \cap Y_2 = \emptyset]\!] & [\textsc{Overlap}] \end{array}$$

Partial Overlap:

$$[\![\mathtt{PO}(Y_1, Y_2)]\!] \quad = \quad [\![\mathtt{O}(Y_1, Y_2)]\!] \wedge \neg [\![\mathtt{P}(Y_1, Y_2)]\!] \wedge \neg [\![\mathtt{P}(Y_2, Y_1)]\!]$$



For all RCC8D formulas F the following holds:
F holds in an adjacency model $\mathcal{M}$ if and only if $\mathcal{M}, X \models_C [\![F]\!]$.

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