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Greedy Algorithms for Tracking Mobile Users in Special Mobility Graphs

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Abstract

An important issue in wireless networks is the design and analysis of strategies for tracking mobile users. Several strategies have been proposed that aim at balancing the cost of updating the user position and the cost of locating a mobile user. The recently proposed reporting center strategy partitions the cellular network into reporting and non-reporting cells, and associates with each reporting cell a set of non-reporting cells, called its vicinity. The users report their position only when they visit a reporting cell. When a call arrives, the user is searched for only in the vicinity of the last visited reporting center. For a given constant Z, the reporting center problem asks for a set of reporting cells of minimum cardinality such that each selected cell has a vicinity of size at most Z so that the update cost is minimized and the locating cost is bounded by Z. The problem was shown to be NP-hard for arbitrary graphs and $Z \geq 2$.

The main contribution of this work is to propose algorithms to optimally solve the reporting center problem for vicinity 2 on interval graphs and for arbitrary vicinity on proper interval graphs.

Key words: wireless networks; interval graphs; tracking problem

1 Introduction

Wireless networks are designed to provide ubiquitous communication services to a large number of mobile users. For this, wireless networks of high capacity are

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required, whose implementation relies essentially on the infrastructures offered by existing cellular networks. A cellular network involves two distinct levels: a stationary level, consisting of fixed base stations interconnected by a wire line network, and a mobile level consisting of mobile users communicating with the base stations via wireless links. The geographic area within which a mobile user can communicate with a particular base station is referred to as a *cell*. The cells overlap to ensure a level of redundancy in the process of *hand-off*, that is, in transferring a mobile user from one cell to another.

An important issue in wireless networks is the design and analysis of strategies for *tracking* mobile users. In fact, a given mobile user could be anywhere within the area covered by the network when a communication must be established. From the system point of view, the tracking problem involves the design of efficient data structures for manipulating the information about the locations visited by the various mobile users. From the mobile user's point of view, the tracking problem is a matter of deciding how frequently the mobile user should inform the base station of its current location in order to help with the search problem.

One of the naive strategies that a mobile user may adopt to update its position is the *always-update* strategy, in which each mobile user transmits an update message whenever it moves into a new cell. The overhead due to traffic involving update messages is very high, while the page space is confined within one single cell. Yet another intuitive strategy is referred to as *never-update*, in which the mobile users never send update messages regarding their current locations. No overhead is involved in updating the mobile user information, but the page space is the whole network.

Several strategies to balance the updating and paging costs have been proposed [1,2,4–6,10–12]. There are *dynamic* strategies in which the mobile users transmit update messages according to their movements and *static* strategies in which the updates take place at predetermined cells. The dynamic strategies are completely *distributed* in the sense that any cell can be a cell where the mobile user reports its position, while the static schemes require to identify in advance the cells where each user must report its position.

One of the dynamic strategies is the *time-based* scheme [4,11] in which each mobile user reputes its position every T time units, where T is a parameter that can vary from user to user. A mobile is paged by searching all possible cells reachable by the user within the elapsed time from the last known cell. Clearly, in this approach, a penalty is paid for stationary users which burden the system with useless update messages. In this approach, the search space is a function of user mobility.

Another strategy is the *distance-based* scheme [1,2,4,10] in which the mobile is required to track the Euclidean distance from the location of the previous update and initiates a new update if the distance exceeds a specific threshold D. The page space is set to the cells falling in a circular region of radius D. Although the distance would ideally be specified in terms of units such as miles or kilometers, often it can be specified in term of the number of cells between the two positions, since an accurate measure of the distance threshold is not always possible.

A possible implementation of the distance-based scheme is the *move-based* strategy [3,4] in which each mobile user updates its location after crossing M boundary cells, where M is a parameter. A mobile is paged on a circular region of radius M cells. In this case, a penalty is paid for users that get in and out from the same subset of cells all the time. The number of useless update messages can be further reduced if each mobile user updates its position after crossing *M different* cells.

The time-, distance- and move-based schemes have been compared in terms of paging cost with varying update rates on a ring cellular topology [4]. It has been observed that the distance-based scheme is the best. This result is intuitively satisfying because the distance threshold directly puts an upper bound on the location uncertainty.

More recently, a *profile-user* strategy [6] was proposed in which a dictionary of individual user's paths is maintained and used to predict the next position of the user.

Among the static strategies, the *location-area* scheme [12] partitions the cellular network in non-overlapping groups of neighboring cells, called Location Areas. A mobile user must sent an update whenever it crosses a location area boundary. The page space is the location area itself. A drawback is that the update traffic originates only in the boundary cells potentially reducing the communication bandwidth of such cells.

An alternative static approach is the *reporting center* scheme [5] where the mobile users transmit update messages only when entering specific predetermined cells termed *reporting cells (or reporting centers)*. The page space is restricted to the *vicinity* of each reporting center, that is, the set of non-reporting cells reachable from a reporting center without crossing other reporting centers. The reporting cells periodically transmit on a specific channel short messages identifying themselves as reporting centers. By listening to that channel, each mobile user determines whether it is in a reporting cell or not and reports its presence accordingly. At the time of establishing a communication, the mobile user to be tracked is searched for only in the vicinity of the reporting cell in which it reported last.

The problem is to select reporting centers in such a way that both the size of the largest vicinity and the total number of reporting centers are minimized. These are conflicting goals since in order to decrease the size of the vicinities the number of reporting centers must be increased (and vice versa). The approach taken in [5] was to bound the vicinity size, and then to minimize the number of reporting centers.

Formally, let a geographic area be modeled by a *mobility graph* $G = (V, E)$ whose vertices V correspond to the cells in such area and whose edges correspond to pairs of cells that overlap. For a given constant Z and a mobility graph G, the reporting center problem $C(G, Z)$ asks for a minimum cardinality set of reporting cells in G such that for each such cell its vicinity has size at most Z.

Unfortunately, it has been shown that for $Z \geq 2$ and for arbitrary mobility graphs G, $C(G, Z)$ is an *NP*-hard problem [5]. The problem becomes tractable only for special classes of graphs, including rings, grids, unidiagonal grids, and bidiagonal grids, for which optimal or near optimal solutions exist.

The main contribution of this work is to address the reporting center problem for arbitrary vicinity on proper interval graphs and for vicinity $Z = 2$ on interval graphs. The motivation for investigating such mobility graphs is that they can be used to model the stationary level of cellular network deployed in narrow areas including valleys, bridges or highways. We begin by presenting a very natural linear-time greedy algorithm for solving problem $C(G, 2)$ on interval graphs, followed by a mixed greedy and dynamic programming algorithm to solve the reporting center problem case for arbitrary vicinity Z on proper interval graph.

2 Interval Graphs

In this section, we introduce terminology and summarize relevant properties of interval graphs [7,9,8]. For a vertex x of a graph $G = (V, E)$, let $N_G(x)$ denote the set of all the vertices of G adjacent to x and let $x \in N_G(x)$. Moreover, let $d_G(x)$ be the cardinality of $N_G(x)$, n the cardinality of V and $\delta(u, v)$ the distance from vertices u and v, i.e., the number of edges that belong to the shortest path of G between u and v.

A graph $I = (V, E)$ is termed an *interval graph* if it has an *interval representation*. That is, if there is a family of intervals $\{T_v\}_{v \in V}$, one interval for each vertex in V, such that there is an edge between the two vertices $u, v \in G$, if and only if $T_u \cap T_v \neq \emptyset$. A graph $P = (V, E)$ is termed a *proper interval graph* if in its interval representation no interval properly contains another.

Denoting left(T_v), right(T_v), respectively, the left and right endpoint of interval T_v, the *canonical order* $<$ of the vertices of an interval graph is defined as follows [9]:

$$u < v \text{ whenever}$$
$$\text{left}(T_u) < \text{left}(T_v), \text{ or } \text{left}(T_u) = \text{left}(T_v) \text{ and right}(T_u) \leq \text{right}(T_v).$$

Similarly, a *canonical order* $\ll$ for the vertices of a proper interval graph is defined in [8] as follows:

$$u \ll v \text{ whenever } \text{left}(T_u) \leq \text{left}(T_v).$$

In particular, it has been shown in [9,8] that the following properties hold:

- for an interval graph $I = (V,E)$, if $u < v < w$ and $uw \in E$, then $uv \in E$;
- for a proper interval graph $P = (V,E)$, if $u \ll v \ll w$, and $uw \in E$, then $uv \in E$ and $vw \in E$.

Let v_i be the i-th vertex in the canonical order, and let $v_{\text{First[i]}}$ and $v_{Last[i]}$ denote respectively the leftmost and rightmost vertices adjacent to v_i in the canonical order. For every vertex v_i, First$[i] \leq i$ and Last$[i] \geq i$ since adjacency is assumed to be reflexive. It is easy to see that the following properties hold for a proper interval graph $P = (V,E)$:

P1 for each vertex v_i, the vertices $v_{First[i]}, \ldots, v_i$ and $v_i, \ldots, v_{Last[i]}$ form a clique (i.e., a set of pairwise adjacent vertices),
P2 for each pair of vertices v_i, v_j with $i < j$, $v_{Last[i]} \leq v_{Last[j]}$.

Moreover, in an interval graph $I = (V,E)$, the following two properties hold for each vertex v_i:

I1 $v_{First[i]}$ is adjacent to all the vertices to its right up to and including v_i,
I2 v_i is adjacent to all the vertices on its right up to and including $v_{Last[i]}$.

It is worth noting that the converse of I1 is not true. That is, v_i is not adjacent to all the vertices to its left up to $v_{First[i]}$.

3 The Reporting Center Problem

A cellular network consists of several cells, each associated with a base station. As we mentioned before, the *mobility graph* $G = (V,E)$ associated with a cellular network contains a vertex for each network cell and an edge for each pair of cells that overlap.

Let a vertex v be termed a *reporting center* if the user must report its position when visiting the network cell associated with v. Otherwise, v is termed a *non-reporting center*.

Let R be the set of reporting cells and $\overline{R}$ be $V - R$. The *vicinity* $z(v)$ of a reporting center $v \in R$ is the set of vertices including the vertex v itself and the vertices in $\overline{R}$ that are reachable from v through a path containing only non-reporting centers. We let $z(R)$ be the size of the largest vicinity among all the vertices in R. Finally, given a graph $G = (V,E)$ and an integer Z, the *reporting center problem* $C(G,Z)$ [4] involves selecting a *minimum size* set R of reporting centers such that $|z(R)| \leq Z$.

For convenience, we also define the *non-reporting center problem* $\overline{C}(G,Z)$ as the problem of selecting a *maximum size* set $\overline{R}$ of non-reporting centers such that for the remaining vertices $R = V - \overline{R}$, $|z(R)| \leq Z$ holds true. In other words, $\overline{R}$ is a solution of $\overline{C}(G,Z)$ if and only if R is a solution of $C(G,Z)$.

3.1 The Reporting Center Problem for Vicinity 2

Let us first consider the reporting center problem for the special case of vicinity 2 on proper interval graphs.

Lemma 1 *If v_g belongs to $\overline{R}$, then all the vertices of $S = \{v_{g+1}, \ldots, v_{Last[Last[g]]}\}$ belong to R.*

Proof. First, we prove that no other vertex can be selected as non-reporting center among the vertices of S. In fact, adding $v_{Last[g]}$ to $\overline{R}$, the vicinity of all the remaining vertices in S is violated. Similarly, adding any other vertex of S to $\overline{R}$, at least the vertex $v_{Last[g]}$ will have a vicinity of size 3. Therefore, all the vertices of S must belong to R. In addition, only the nodes on S are forced by v_g to belong to R since each node v_x on the right of $v_{Last[Last[g]]}$ and v_g are at distance at least 3 and therefore they cannot belong to the same vicinity. □

Theorem 1 *For proper interval graphs and vicinity 2, the reporting center problem can be solved in time linear in the size of the graph.*

Proof.

Procedure $\overline{C}(P,2)$;
$\{$*Input:* a proper interval graph $P = (V,E)$ with $V = \{v_1, v_2, \ldots, v_n\}$ in canonical order. The array *Last* whose ith entry, $1 \leq i \leq n$, stores the index of the rightmost vertex adjacent to v_i. The sets $\overline{R} = \emptyset$ and $R = V$;
Output: a maximum size non-reporting center set $\overline{R}\}$.

begin
$i \leftarrow 1$;
while $i \leq n$ **do**
  $\overline{R} \leftarrow \overline{R} \cup \{v_i\}$;
  $R \leftarrow R - \{v_i\}$;
  $i \leftarrow \text{Last}[\text{Last}[i]] + 1$;
end

The algorithm traverses the vertices in canonical order from left to right, and selects the vertices that belong to $\overline{R}$ according to the greedy choice suggested by Lemma 1:

the new entry v_g in $\overline{R}$ is the leftmost node in the canonical order not yet forced to belong to R by the vertices already in $\overline{R}$ (if any). Moreover, all the nodes

$$\{v_{g+1}, \ldots, v_{Last[Last[g]]}\}$$

are forced in R.

The correctness of the greedy solution follows from Lemma 1. It remains to prove its optimality. The first time the greedy choice is applied v_1 is selected. Let us first prove that there always exists an optimal solution that begins with the greedy choice. Let T be an optimal solution for $\overline{C}(P,2)$, and assume without loss of generality the vertices of T sorted by increasing subscripts. Let the first vertex in T be v_k. If $k >$ Last[Last[1]], since $v_1, \ldots, v_{k-1}$ are not in T, we can immediately claim that $T \cup \{v_1\}$ is a larger solution for $\overline{C}(P,2)$, contradicting the optimality of T. Then, let the first vertex in T be v_k, with $1 < k \leq$ Last[Last[1]]. Since $k > 1$, by Property P2, $Last[k] \geq Last[1]$, and Last[Last[k]] $\geq$ Last[Last[1]]. Hence, the greedy choice v_1 can substitute v_k, and the new solution $B = (T - \{v_k\} \cup \{v_1\})$ is correct and optimal as it was T.

Now, once the greedy choice of v_1 has been made, the problem reduces to solve the non-reporting center problem on $\overline{C}(P', 2)$, with $P' = (V', E')$ and $V' = V - \{v_i | 1 \leq i \leq Last[Last[1]]\}$. Clearly, $T' = T - v_k$ is an optimal solution for such a problem. In fact, if we could find a better solution B' for such a problem, adding vertex v_1 to B' would yield to an optimal B with more vertices than T, thereby contradicting the optimality of T. That is, after each greedy choice has been done, we are left with an optimization problem of the same form as the original one. Then, by induction on the number of the choices done, accomplishing the greedy choice at every step produces an optimal solution.

Finally, it is easy to see that the time complexity of the algorithm is linear in the size of the vertex set V of P. □

In the remainder of the section, we devise a linear-time algorithm for the reporting center problem for vicinity 2 on (general) interval graphs. For each vertex v_g, define $Furthest[g]$ as the subscript of the rightmost vertex at distance at most 2 from v_g in the canonical order.

Lemma 2 *If v_g belongs to $\overline{R}$, v_g forces in R, among the vertices to its right, all vertices $S = \{v_{g+1}, \ldots, v_{Furthest[g]}\}$ and only them.*

Proof. First, suppose that another vertex among the vertices of S can be selected as non-reporting center. Let v_w be the adjacent vertex of v_g such that $Last[w] = Furthest[g]$. Note that by adding vertex v_w to $\overline{R}$, at least the vertex $v_{Last[w]}$ will have a vicinity of size 3. Similarly, by adding another vertex of S, but v_w, to $\overline{R}$, at least the vertex v_w will have a vicinity of size 3. Therefore, all the vertices of S must belong to R. Finally, since any node v_x to the right of $v_{Furthest[g]}$ and v_g are at least at distance 3, v_g and v_x cannot belong to the same vicinity. Therefore, only the nodes of S are forced by v_x into R. □

Theorem 2 *For interval graphs and vicinity 2, the reporting center problem can be solved in time linear in the size of the graph.*

Proof.

Procedure $\overline{C}(I,2)$;
$\{$*Input:* an interval graph $I = (V,E)$ with $V = \{v_1, v_2, \ldots, v_n\}$ in canonical order. The sets $\overline{R} = \emptyset$ and $R = V$;
Output: a maximum size set $\overline{R}\}$.

```
begin
for  i ← 1 to n              do
        compute Last[i];
for  i ← 1 to n              do
        Furthest[i]  ← Last[i];
        for  vj ∈ NG(vi)      do
                Furthest[i]  ← max{Furthest[i], Last[j]}
t ← 1; t' ← 1;
m ← Furthest[1]
while    t ≤ n               do
        for  vj ∈ NG(vt)      do
                if Furthest[j] < m      then
                                    m ← Furthest[j];
                                    t' ← j;
        R̄ ← R̄ ∪ {t'};
        R ← R − {t'};
        t ← m+1; t' ← m+1;
        m ←  Furthest[t];
end
```

The correctness follows from Lemma 2. To argue for the optimality of the greedy solution, let T be an optimal solution for $\overline{C}(I,2)$, and assume the vertices of T sorted by increasing subscripts. Let v_k be the leftmost vertex in T, with $k \leq Furthest[1]$. Indeed, if $k > Furthest[1]$, we can immediately claim that $T \cup \{v_1\}$ is a larger solution for $\overline{C}(I,2)$, contradicting the optimality of T. Then, let v_g be the first greedy choice, and let w be the node adjacent to v_g such that $Last[w] = Furthest[g]$.

We now claim that if $v_k \neq v_g$, with $1 \leq k \leq Furthest[g]$, then $Furthest[k] \geq Furthest[g]$. Indeed consider the following two cases:

(1) if $v_k \in N_G(v_1)$, then, by the greedy choice definition, $Furthest[k] \geq Furthest[g]$;

(2) if $v_k \in \{v_{Last[1]+1}, \ldots, v_{Furthest[g]}\}$, then, by definition of v_g, $Furthest[1] \geq Furthest[g]$. So there is a node $v_{w_1} \in N_G(v_1)$ with $Last[w_1] = Furthest[1] \geq Furthest[g]$. Hence, $v_k \in N_G(v_{w_1})$, and $Furthest[k] \geq Last[w_1] \geq Furthest[g]$.

In conclusion, since $Furthest[k] \geq Furthest[g]$, v_k can be replaced by v_g achieving a new solution $B = (T - \{v_k\} \cup \{v_g\})$, which is correct and optimal. Moreover, once the greedy choice of v_g has been done, the problem reduces to solve the non-reporting center problem $\overline{C}(I',2)$, with $I' = (V',E')$, and $V' - \{v_i | 1 \leq i \leq Furthest[g]\}$. Clearly, $T' = T - \{v_k\}$ is an optimal solution for $\overline{C}(I',2)$. If not, we could find a better solution B' for such a problem and adding the vertex v_g to B', we would form an optimal solution B with cardinality larger than S, a contradiction.

That is, after each greedy choice has been made, we are left with an optimization problem of the same form as the original one. Then, by induction on the number of the choices made, the greedy choice produces an optimal solution. □

3.2 The Reporting Center Problem for Arbitrary Vicinity on Proper Interval Graphs

In this section, we propose a solution for the reporting center problem on proper interval graphs for arbitrary vicinity Z.

Given a proper interval graph $P = (V,E)$ and a set of vertices $N \subset V$, denote by $P(N) = (N,E')$ the subgraph of P obtained by restricting V to N and E to the edges E' between pairs of vertices of N, i.e. $E' = \{uv | \ u,\ v \ \in \ N, \text{ and } \ uv \in E\}$. The set of vertices N is termed a *connected set* if the subgraph $P(N)$ is connected.

Lemma 3 *Let T be a solution of $\overline{C}(P,Z)$ which has selected as non-reporting centers the connected set $N = \{v_{i_1}, v_{i_2}, \ldots, v_{i_{Z-1}}\}$. Then all the vertices of the set*

$S = \{v_{i_{Z-1}+1}, \ldots, v_{Last[i_{Z-1}]}, \ldots, v_{Last[Last[i_{Z-1}]]}\}$ to the right of $v_{i_{Z-1}}$ are forced in R by N.

Proof. First, we argue that no vertex of S can be selected as non-reporting center. In fact, suppose that $v_{Last[i_{Z-1}]}$ is added to N. Then, the vicinity of all the remaining nodes of S will larger than Z. Similarly, adding to N any other node of S, but $v_{Last[i_{Z-1}]}$, at least the vicinity of $v_{Last[i_{Z-1}]}$ will be larger than Z. Therefore, all the vertices of S must belong to R. Moreover, the nodes to the right of the rightmost node of S can reach the non-reporting centers in N only by a path containing at least two nodes of S. In other words, N and a node to the right of the rightmost node of S cannot belong to the same vicinity. Thus, N forces only the nodes of S in R. □

Lemma 4 *For $1 \leq k \leq Z-1$, an optimal solution of $\overline{C}(P,Z)$ must select* at least k

non-reporting centers among the vertices $v_1, \ldots, v_{Last[Last[k]]}$.

Proof. Suppose that there is an optimal solution B which selects, for a given k with $1 \le k \le Z-1$, only $j < k$ non-reporting centers among the vertices $v_1, \ldots, v_{Last[Last[k]]}$. Let $N' = \{v_{i_1}, v_{i_2}, \ldots, v_{i_j}\}$ be such selected non-reporting centers. By substituting the vertices $N = \{v_1, \ldots, v_k\}$ for N', the new solution is still correct. Indeed, the vicinity of the vertices on the right of $v_{Last[Last[k]]}$ does not increase since each node v_w, with $w > Last[Last[k]]$, reaches whatever node in N only by a path containing at least two reporting center vertices. Moreover, the vicinity of each node $v_{k+1}, \ldots, v_{Last[Last[k]]}$ becomes $k+1$, but since $k+1 \le Z$, the vicinity constraint is not violated. Therefore, there is a new solution $T' = B - N' \cup \{v_1, \ldots, v_k\}$ which has larger size than B, contradicting that B was an optimal solution for $\overline{C}(P,Z)$. □

A connected set is said to be a *maximal connected set* if no connected set contains it properly. From the previous lemma, the following claim follows:

Lemma 5 *If there is an optimal solution T for the problem $\overline{C}(P,Z)$ whose leftmost maximal connected set $N = \{v_{i_1}, \ldots, v_{i_k}\}$ of non-reporting centers has size k, with $1 \le k \le Z-1$ and with $i_k \le Last[Last[k]]$, then $T' = T - N \cup \{v_1, v_2, \ldots, v_k\}$ is an optimal greedy solution.*

For $1 \le k \le Z-1$, note that an optimal solution can choose more than k non-reporting centers among the vertices $v_1, \ldots, v_{Last[Last[k]]}$ and that the greedy choice $\{v_1, v_2, \ldots, v_k\}$ can be applied only when exactly k vertices are chosen.

Lemma 6 *Given two maximal connected sets S_1 and S_2 of non-reporting centers, with $S_1 = \{v_{i_1}, v_{i_2}, \ldots, v_{i_j}\}$, and $S_2 = \{v_{i_{j+1}}, v_{i_{j+2}}, \ldots, v_{i_{j+k}}\}$, such that the distance $\delta(v_{i_{j+1}}, v_{i_j})$ between v_{i_j} and $v_{i_{j+1}}$ is 2, S_1 and S_2 may belong to the same optimal solution T of $\overline{C}(P,Z)$ if and only if $j+k \le Z-1$. Moreover, if there is an optimal solution T containing both S_1 and S_2, let $S'_1 = \{v_1, v_2, \ldots, v_j\}$ and $S'_2 = \{v_{Last[j]+1}, v_{Last[j]+2}, \ldots, v_{Last[j]+k}\}$, then $T' = T - S_1 - S_2 \cup S'_1 \cup S'_2$ is an optimal solution too.*

Proof. Let v_w be the vertex adjacent to both v_{i_j} and $v_{i_{j+1}}$. The vicinity of v_w is equal to $j+k$. Then, S_1 and S_2 can belong to the same solution T, if and only if $j+k \le Z-1$.

Moreover, if there is an optimal solution T that contains both S_1 and S_2, substituting S'_1 and S'_2 for, respectively, S_1 and S_2, since the distance between the rightmost vertex in S_1 and the leftmost vertex in S_2 is still 2 and since $Last[i_j] \ge Last[j]$ and $Last[i_{j+k}] \ge Last[Last[j]+k]$ by Property P2, the vicinity of the reporting centers in T on the right of $v_{Last[i_{j+k}]}$ cannot increase. So, $T' = T - S_1 - S_2 \cup S'_1 \cup S'_2$ is an optimal solution for $\overline{C}(P,Z)$. □

In order to state the main result, let $P_j = (V_j, E_j)$ stand for the subgraph of $P = (V, E)$ with $V_j = \{v_j, \ldots, v_n\}$ and $E_j = \{e = (v_m, v_n) | e \in E, \text{ and } m, n \geq j\}$. Let $\overline{R}[j]$ be an optimal solution for $\overline{C}(P_j, Z)$. Clearly, $P_1 = P$ and $\overline{R} = \overline{R}[1]$.

Moreover, let the *k-non-reporting center problem* $\overline{C}_k(P_j, Z)$ select the maximum size set $\overline{R}[j,k]$ of non-reporting centers in P_j in such way that

(1) the vicinity of each reporting center in $R[j,k] = V - \overline{R}[j,k]$ is less than Z, and
(2) the size of the leftmost maximal connected set of non-reporting centers of $\overline{R}[j,k]$ is exactly equal to k.

Lemma 5 showed that if there is an optimal solution that begins with a maximal connected set of cardinality k, then there is another optimal solution for $\overline{C}(P,Z)$ that selects the leftmost k vertices of V. Let us call such an optimal solution a *k-greedy solution*.

We do not know in advance the cardinality of the leftmost maximal connected set of $\overline{R}$, but it must be:

$$\overline{R} = \max_{1 \leq k \leq Z-1} \overline{R}[1,k].$$

In addition, according to Lemmas 5 and 6, the k-greedy solution of $\overline{C}_k(P,Z)$ that begins with the maximal connected set of size $v_1, v_2, \ldots, v_k$, is completed either by the solution $\overline{R}[Last[Last[k]]+1]$ of $\overline{C}(P_{Last[Last[k]]+1}, Z)$ or by the largest solution among the solutions $\overline{R}[Last[k]+1, i]$ for $\overline{C}_i(P_{Last[k]+1}, Z)$, for $1 \leq i \leq Z-1-k$. Formally,

$$\overline{R}[1,k] = \{v_1, \ldots, v_k\} \cup \max\left(\overline{R}[Last[Last[k]]+1], \max_{1 \leq i \leq Z-1-k} \overline{R}[Last[k]+1, i]\right). \tag{1}$$

Hence, after each greedy choice has been made, we are left with optimization problems of smaller size to be solved.

For example, $\overline{C}(P,3)$, $\overline{R} = \max_{1 \leq k \leq 2} \overline{R}[1,k]$. Applying Equation 1, $\overline{R}$ is the non-reporting set with largest cardinality among the following three solutions:

(1) $\{v_1 \cup \overline{R}[Last[Last[1]]+1]\}$,
(2) $\{v_1, v_2 \cup \overline{R}[Last[Last[2]]+1]\}$, and
(3) $\{v_1 \cup \overline{R}[Last[1]+1, 1]\}$.

In Figure 1, the non-reporting center problem for vicinity 3 is solved on the proper interval graph P with 7 vertices applying three times the third rule above.

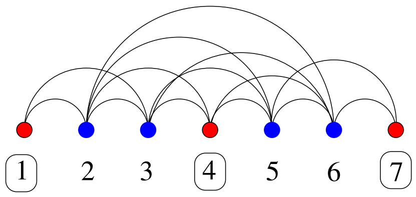

Fig. 1. An optimal solution for $\overline{C}(P,3)$ which selects the cells $\{1,4,7\}$ as non-reporting centers.

Theorem 3 *For proper interval graphs and vicinity Z, with $Z \geq 3$, the reporting center problem can be solved in time $O(nZ^2)$ and $O(nZ)$ space, where n is the size of the graph vertex-set V.*

Proof.

Procedure $\overline{C}(P,Z)$;
$\{$*Input:* a proper interval graph $P=(V,E)$ with $V=\{v_1,v_2,\dots,v_n\}$ in canonical order, with $n \geq Z$. The array `Last` whose r-th entry will store the subscript of the rightmost vertex adjacent to r. The matrix `sol` of size $n \times (Z-1)$. Each entry $[j,k]$ of `sol`, initialized to $\emptyset$, will store $\overline{R}[j,k]$.
Output: a maximum size non-reporting center set $\overline{R}$ for vicinity Z;

1 **begin**
2 **for** $j \leftarrow 1$ **to** n **do**
3 compute Last$[j]$;
4 **for** $k \leftarrow 1$ **to** $Z-1$ **do**
5 $sol[j,k] \leftarrow \emptyset$;
6 **for** $j \leftarrow n$ **to** $n-Z+2$ **do**
7 **for** $k \leftarrow 1$ **to** $n-j+1$ **do**
8 $sol[j,k] \leftarrow \{n-k+1,\dots,n\}$;
9 **for** $j \leftarrow n-Z+1$ **downto** 1 **do**
10 **for** $i \leftarrow 1$ **to** $Z-1$ **do**
11 $w \leftarrow Last[Last[j+i-1]]+1$;
12 $y \leftarrow Last[j+i-1]+1$;
13 $m1 \leftarrow \max_{1 \leq k \leq Z-1} sol[w,k]$; /* $m1 = \overline{R}[w]$ */
14 **if** $j = Z-1$ **than** $m2 \leftarrow \emptyset$
15 **else** $m2 \leftarrow \max_{1 \leq k \leq Z-j-1}\{sol[y,k]\}$;
16 $sol[j,i] \leftarrow \{j,j+1,\dots,j+i-1\} \cup \max\{m1,m2\}$;
17 $\overline{R} \leftarrow \max_{1 \leq k \leq Z-1}(sol[1,k])$;
18 **end**

In lines 6-8 of the above procedure, $\overline{R}[j,k] = \{v_j,\dots,v_n\}$ is directly stored in $sol[j,k]$ In fact, since the number of vertices of the considered subgraph is smaller than k, i.e. $n-j+1 \leq k$, all the vertices of P_j are selected as non-reporting centers.

In lines 9-16, the solution of the remaining entries of `sol` are computed according to Equation 1.

Having proved in Lemmas 5 and 6 that for each optimal solution there is an equivalent greedy solution, the optimality follows by induction. □

Note that the above algorithm implicitly solves the reporting center problem for each subgraph of P. In fact, for any subgraph P_j, with $Z \leq j \leq n$, the solution $\overline{R}(j)$ of $\overline{C}(P_j, Z)$ is simply computed as:

$$\overline{R}(j) = \max_{1 \leq k \leq Z-1} sol[j,k].$$

This aspect is specially relevant whenever we assume a dynamic proper interval graph P. In fact, by adding or deleting a node of P, we do not need to compute from scratch the final new solution for the modified graph P since some of the partial results computed in the matrix `sol` are still valid.

4 Conclusion

We have studied the problem of tracking mobile user in wireless networks with the goal of trading the cost of updating the user location with the cost of locating a mobile user. The reporting center problem is solved optimally for arbitrary vicinity Z in time $O(|V|Z^2)$ on proper interval graphs. For interval graphs, the reporting center problem is solved optimally only for the case of vicinity 2 in time $O(|V|)$. It remains an open question whether the reporting center problem can be solved in polynomial time on general interval graphs for arbitrary vicinities.

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